

# Measures of Center

A **measure of center** is a measure that represents the center, or typical value, of a data set. The *mean*, *median*, and *mode* are measures of center.

Mean	Median	Mode
The <b>mean</b> of a numerical data set is the sum of the data divided by the number of data values. The symbol $\bar{x}$ represents the mean. It is read as “x-bar.”	The <b>median</b> of a numerical data set is the middle number when the values are written in numerical order. When a data set has an even number of values, the median is the mean of the two middle values.	The <b>mode</b> of a data set is the value or values that occur most often. There may be one mode, no mode, or more than one mode. Mode is the only measure of center that can represent a nonnumerical data set.

**Example 1** The table shows the sizes (in kilobytes) of emails in your inbox.

Email Sizes (kilobytes)				
1.5	13	1.8	1.9	9.1
2.4	2.8	9.2	2	11
5.6	5	4.9	5.5	11

- Find the mean, median, and model of the email sizes.
- Which measure of center best represent the data? Explain.

a. **Mean**  $\bar{x} = \frac{1.5 + 13 + 1.8 + \dots + 5.5 + 11}{15} = 5.78$

**Median** 1.5, 1.8, 1.9, 2, 2.4, 2.8, 4.9, **5**, 5.5, 5.6, 9.1, 9.2, 11, 11, 13  
↑  
middle value

**Mode** 1.5, 1.8, 1.9, 2, 2.4, 2.8, 4.9, 5, 5.5, 5.6, 9.1, 9.2, 11, 11, 13 **11 occurs most often.**

► The mean is 5.78 kilobytes, the median is 5 kilobytes, and the mode is 11 kilobytes.

- The median best represents the data. The mean and mode are both greater than most of the data.

## Practice

Check your answers at [BigIdeasMath.com](http://BigIdeasMath.com).

Find the mean, median, and mode of the data set.

- 35, 44, 40, 35, 54, 50
- 14, 8, 10, 12, 13, 18, 6, 11, 16
- 834, 654, 711, 590, 578, 861, 525
- 4, 8, 5, 6, 4, 5, 4, 2, 6, 5, 4, 3, 5, 4, 6, 5
- 0.6, 1.4, 0.7, 2, 1.5, 1.2, 1.4, 0.9, 0.7, 1.8
- $7\frac{3}{4}$ ,  $8\frac{1}{2}$ , 8,  $6\frac{3}{4}$ ,  $7\frac{3}{4}$ , 8,  $8\frac{1}{4}$ , 8

- APARTMENTS** The table shows the monthly rental prices for apartments in a city. Find the mean, median, and mode of the prices. Which measure of center best represents the data? Explain.

Monthly Rental Prices			
\$535	\$625	\$850	\$480
\$895	\$420	\$500	\$485
\$1175	\$490	\$510	\$550

# Mean Absolute Deviation

A **mean absolute deviation** is an average of how much data values differ from the mean.

**Finding the Mean Absolute Deviation (MAD)**

**Step 1** Find the mean of the data.

**Step 2** Find the distance between each data value and the mean.

**Step 3** Find the sum of the distances in Step 2.

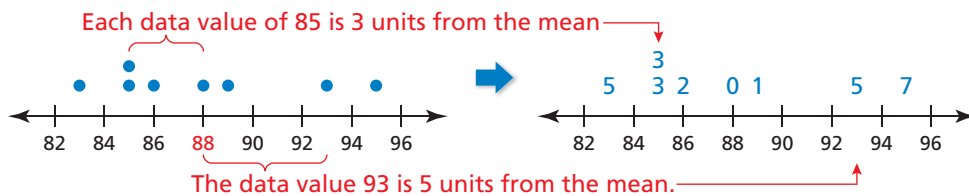
**Step 4** Divide the sum in Step 3 by the total number of data values.

**Example 1** The scores on a science test are shown below. Find and interpret the mean absolute deviation of the data.

86, 93, 88, 85, 89, 95, 85, 83

**Step 1** Mean =  $\frac{86 + 93 + 88 + 85 + 89 + 95 + 85 + 83}{8} = \frac{704}{8} = 88$

**Step 2** You can use a dot plot to organize the data. Replace each dot with its distance from the mean.



**Step 3** The sum of the distances is  $5 + 3 + 3 + 2 + 0 + 1 + 5 + 7 = 26$ .

**Step 4** The mean absolute deviation is  $\frac{26}{8} = 3.25$ .

► So, the data values differ from the mean by an average of 3.25 points.

## Practice

*Check your answers at [BigIdeasMath.com](http://BigIdeasMath.com).*

**Find and interpret the mean absolute deviation of the data.**

1. 9, 10, 11, 11, 12, 12, 13, 13, 14, 15
2. 2, 4, 4, 5, 6, 7, 7, 7
3. 24, 26, 27, 27, 28, 28, 30, 32
4. 8, 28, 29, 31, 32, 35, 38, 41, 43, 44

5. **TEMPERATURES** The table shows the high temperatures for several July days in a city. Find and interpret the mean absolute deviation of the data.

Temperatures (°F)				
85	79	82	80	90
79	83	83	78	78
80	82	82	86	83

# Standard Deviation

**Standard Deviation**

The **standard deviation** of a numerical data set is a measure of how much a typical value in the data set differs from the mean. The symbol  $\sigma$  represents the standard deviation. It is read as “sigma.” It is given by

$$\sigma = \sqrt{\frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \cdots + (x_n - \bar{x})^2}{n}}$$

where  $n$  is the number of values in the data set. The deviation of a data value  $x$  is the difference of the data value and the mean of the data set,  $x - \bar{x}$ .

**Step 1** Find the mean,  $\bar{x}$ .

**Step 2** Find the deviation of each data value,  $x - \bar{x}$ .

**Step 3** Square each deviation,  $(x - \bar{x})^2$ .

**Step 4** Find the mean of the squared deviations. This is called the *variance*.

**Step 5** Take the square root of the variance.

**Example 1** The data set shows the hourly pay rates for several employees at a restaurant. Find and interpret the standard deviation of the data set.

**\$9.30, \$8.00, \$7.50, \$8.50, \$10.40, \$9.30, \$9.70, \$9.30**

**Step 1** Find the mean,  $\bar{x}$ .  $\bar{x} = \frac{9.3 + 8 + \cdots + 9.7 + 9.3}{8} = \frac{72}{8} = 9$

**Step 2** Find the deviation of each data value,  $x - \bar{x}$ , as shown.

**Step 3** Square each deviation,  $(x - \bar{x})^2$ , as shown.

**Step 4** Find the mean of the squared deviations, or variance.

$$\frac{0.09 + 1 + \cdots + 0.49 + 0.09}{8} = \frac{6.22}{8} \approx 0.8$$

**Step 5** Use a calculator to take the square root of the variance.

$$\sqrt{\frac{6.22}{8}} \approx 0.9$$

$x$	$\bar{x}$	$x - \bar{x}$	$(x - \bar{x})^2$
9.3	9	0.3	0.09
8	9	-1	1
7.5	9	-1.5	2.25
8.5	9	-0.5	0.25
10.4	9	1.4	1.96
9.3	9	0.3	0.09
9.7	9	0.7	0.49
9.3	9	0.3	0.09

► The standard deviation is about 0.9. This means that the typical hourly pay rate of an employee differs from the mean by about \$0.90.

## Practice

*Check your answers at BigIdeasMath.com.*

**Find and interpret the standard deviation of the data set.**

1. Exam scores: 98, 95, 82, 85, 77, 85, 91, 93, 75, 80, 81, 90
  
2. Stock prices per share: \$9.70, \$13.50, \$9.50, \$7, \$7.80, \$16.40, \$10.20, \$9, \$14.90, \$12

# Venn Diagrams

A **Venn diagram** uses shapes to describe relationships between two or more sets.

**Example 1** Draw a Venn diagram of the positive integers less than 19, where set *A* consists of factors of 18 and set *B* consists of even numbers.

Positive integers less than 19:

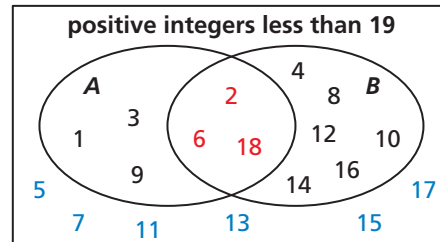
1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18

Set *A* (factors of 18): 1, 2, 3, 6, 9, 18

Set *B* (even numbers): 2, 4, 6, 8, 10, 12, 14, 16, 18

Both set *A* and set *B*: 2, 6, 18

Neither set *A* nor set *B*: 5, 7, 11, 13, 15, 17



**Example 2** Use the Venn diagram above to decide whether the statement is *true* or *false*. Explain your reasoning.

- If a positive integer less than 19 is not even, then it is not a factor of 18.
  - ▶ False. 1, 3, and 9 are not even, but they are factors of 18.
- All positive integers less than 19 that are even are factors of 18.
  - ▶ False. 4, 8, 10, 12, 14, and 16 are even, but they are not factors of 18.

## Practice

Check your answers at [BigIdeasMath.com](http://BigIdeasMath.com).

Draw a Venn diagram of the sets described.

- Of the positive integers less than 13, set *A* consists of the factors of 12 and set *B* consists of even numbers.
- Of the positive integers less than 11, set *A* consists of prime numbers and set *B* consists of odd numbers.
- Of the positive integers less than 25, set *A* consists of the multiples of 3 and set *B* consists of the multiples of 4.

Use the Venn diagrams you drew in Exercises 1–3 to decide whether the statement is *true* or *false*. Explain your reasoning.

- The only positive factors of 12 less than 13 that are not even are 1 and 3.
- All positive odd numbers less than 11 are prime.
- All prime numbers less than 11 are odd.
- There are 2 positive integers less than 25 that are both a multiple of 3 and a multiple of 4.
- The number of positive integers less than 25 that are multiples of either 3 or 4 is equal to the number of positive integers less than 25 that are not multiples of either 3 or 4.

# Dot Plots

The **dot plot** uses a number line to show the number of times each value in a data set occurs. Dot plots (or *line plots*) show clusters, peaks, and gaps in a data set. You can also use a dot plot to identify the shape of a distribution.

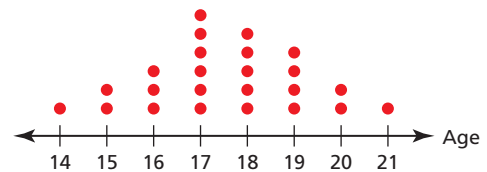
Uniform	Symmetric	Skewed
<p>All the dots are about the same height.</p> <p>A uniform distribution is also symmetric.</p>	<p>The data on the right of the distribution are approximately a mirror image of the data on the left of the distribution.</p>	<p>The "tail" extends either left or right. A distribution is <i>skewed left</i> when most of the data are on the right and <i>skewed right</i> when most of the data are on the left.</p>

**Example 1** The table shows the ages of volunteers participating in a park cleanup. Draw a dot plot that represents the data. Describe the distribution.

Ages								
19	17	18	17	18	18	20	19	
15	17	16	17	16	17	18	19	
16	18	19	20	15	17	21	14	

Draw a number line that includes the least value, 14, and the greatest value, 21. Then place a dot above the number line for each data value.

▶ A peak occurs at 17. The data on the right of the distribution are approximately a mirror image of the data on the left of the distribution. So, the distribution is symmetric.



## Practice

Check your answers at [BigIdeasMath.com](http://BigIdeasMath.com).

Draw a dot plot that represents the data. Describe the distribution.

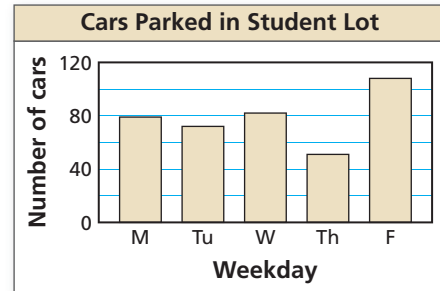
- Televisions in households:  
2, 4, 3, 6, 2, 1, 3, 4, 0, 1, 3, 3, 2, 2, 5, 2, 1, 7, 5, 4
- Ages of new drivers:  
15, 16, 16, 17, 16, 15, 15, 18, 16, 17, 18, 18, 15, 17, 17, 18
- Heights of basketball players (in inches):  
74, 79, 80, 81, 71, 73, 73, 72, 78, 79, 80, 79, 72, 73

# Bar Graphs and Line Graphs

A **bar graph** shows data in specific categories. A **line graph** shows how data change over time.

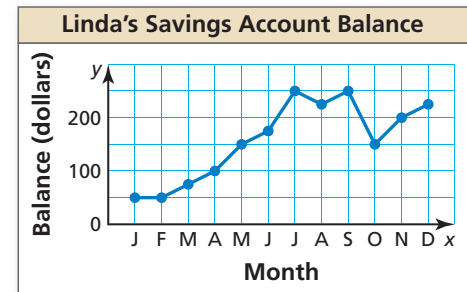
**Example 1** Use the bar graph to answer each question.

- a. On which day of the week were the greatest number of cars parked in the student lot?
- ▶ The tallest bar on the graph is the bar for Friday. So, the answer is Friday.
- b. How many cars were parked in the student lot on Monday?
- ▶ The bar for Monday shows that about 80 cars were parked in the student lot.



**Example 2** Use the line graph to answer each question.

- a. In which month(s) was Linda's account balance \$150?
- ▶ From the graph, Linda's account balance was \$150 in May and October.
- b. Between which two consecutive months did Linda's account balance increase the most?
- ▶ Of the graph's line segments that have positive slopes, the graph is steepest from June to July. So, Linda's account balance increased the most between June and July.



## Practice

Check your answers at [BigIdeasMath.com](http://BigIdeasMath.com).

Use the bar graph in Example 1 to answer the question.

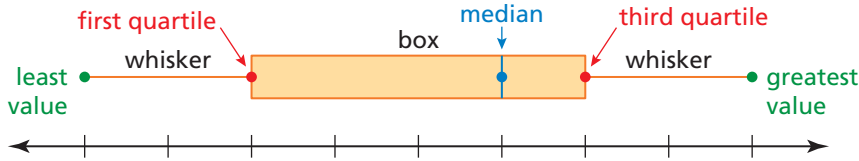
- On which day of the week were the least number of cars parked in the student lot?
- On which day(s) of the week were there about 70 cars parked in the student lot?
- About how many more cars were parked in the student lot Friday than on Thursday?
- About how many more cars were parked in the student lot on Friday than on Monday?

Use the line graph in Example 2 to answer the question.

- In which month(s) was Linda's account balance \$250?
- Between which two consecutive months did Linda's account balance decrease the most?
- How much less was Linda's account balance in October than in July?
- How much more was Linda's account balance in September than in April?

# Box-and-Whisker Plots

A **box-and-whisker plot** represents a data set along a number line by using the least value, the greatest value, and the quartiles of the data. The five numbers that make up the box-and-whisker plot are called the **five-number summary** of the data set.



You can use a box-and-whisker plot to identify the shape of a distribution.

Skewed Left	Symmetric	Skewed Right
The left whisker is longer than the right whisker. Most of the data are on the right side of the plot.	The whiskers are about the same length. The median is in the middle of the plot.	The right whisker is longer than the left whisker. Most of the data are on the left side of the plot.

**Example 1** The data set shows the prices (in dollars) of lacrosse helmets. Make a box-and-whisker plot that represents the data. Describe the distribution.

125, 120, 250, 110, 190, 220, 145, 260, 240, 150, 170, 200

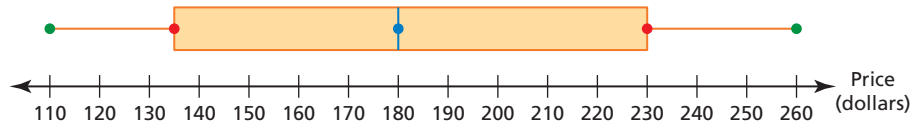
**Step 1** Order the data. 110 120 125 145 150 170 190 200 220 240 250 260

Find the median and the quartiles.

first quartile, 135      median, 180      third quartile, 230

**Step 2** Draw a number line that includes the least value, 110, and the greatest value, 260. Graph points above the number line that represent the five-number summary.

**Step 3** Draw a box using the quartiles. Draw a line through the median. Draw whiskers from the box to the least and the greatest values.



► The whiskers are about the same length and the median is in the middle of the plot. So, the distribution is symmetric.

## Practice

Check your answers at [BigIdeasMath.com](http://BigIdeasMath.com).

Make a box-and-whisker plot that represents the data. Describe the distribution.

- Video game prices (in dollars):  
45, 40, 50, 35, 30, 40, 40, 30, 45, 60
- Exam scores: 79, 86, 100, 82, 94, 98, 96,  
86, 90, 92, 62, 84

# Circle Graphs

A **circle graph** displays data as sections of a circle. The entire circle represents all of the data. Each section represents part of the data and can be labeled using the actual data or the data expressed as fractions, decimals, or percents. When the data are expressed as fractions or decimals, the sum of the data is 1. When the data are expressed as percents, the sum of the data is 100%.

The sum of the angle measures in a circle graph is  $360^\circ$ . When the data are given as percents, multiply the decimal form of each percent by  $360^\circ$  to find the angle measure for each section.

**Example 1** The table shows the results of a survey.  
Display the data in a circle graph.

Favorite Season	Students
Winter	16
Spring	29
Summer	67
Fall	48

**Step 1** Find the total number of students in the survey.

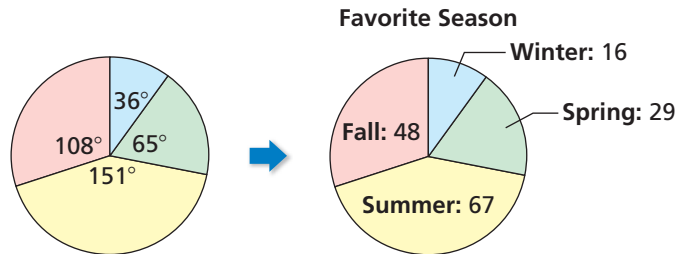
$$16 + 29 + 67 + 48 = 160$$

**Step 2** Find the angle measure for each section of the circle graph.  
Multiply the fraction of students that chose each season by  $360^\circ$ .

**Winter:**  $\frac{16}{160} \cdot 360^\circ = 36^\circ$       **Spring:**  $\frac{29}{160} \cdot 360^\circ \approx 65^\circ$

**Summer:**  $\frac{67}{160} \cdot 360^\circ \approx 151^\circ$       **Fall:**  $\frac{46}{160} \cdot 360^\circ = 108^\circ$

**Step 3** Use a protractor to draw the angle measures found in Step 2 on a circle. Then label the sections.



## Practice

Check your answers at [BigIdeasMath.com](http://BigIdeasMath.com).

Display the data in a circle graph.

1.

Candidate	Ben	Emma	Elle	Ryan	Li
Votes	94	172	113	161	60

2.

Transportation	Percent of Students
Car	24%
Bus	20%
Bike	13%
Walk	43%

# Stem-and-Leaf Plots

A **stem-and-leaf plot** uses the digits of data values to organize a data set. Each data value is broken into a **stem** (digit or digits on the left) and a **leaf** (digit or digits on the right).

A stem-and-leaf plot shows how data are distributed.

Stem	Leaf
1	0 0 1 3 8 9
2	1 5 8
3	6
4	1 7

The key explains what the stems and leaves represent.

Key: 1|0 = 10

## Example 1 Make a stem-and-leaf plot of the data.

**Step 1** Order the data.

12, 14, 15, 16, 16, 18, 18, 19, 23, 24, 24, 25, 28, 29, 30, 30, 30, 31, 32, 34, 35, 36, 39, 68

**Step 2** Choose the stems and the leaves. Because the data values range from 12 to 68, use the *tens* digits for the stems and the *ones* digits for the leaves. Be sure to include the key.

Ages of Actors in a Play					
30	24	24	35	12	15
18	31	30	30	19	32
18	36	16	28	39	16
68	29	34	23	25	14

**Step 3** Write the stems to the *left* of the vertical line.

**Step 4** Write the leaves for each stem to the *right* of the vertical line.

Order the stems vertically.

Include stems without leaves.

Ages of Actors in a Play

Stem	Leaf
1	2 4 5 6 6 8 8 9
2	3 4 4 5 8 9
3	0 0 0 1 2 4 5 6 9
4	
5	
6	8

Write the leaves horizontally.

Key: 1|2 = 12

## Practice

Check your answers at [BigIdeasMath.com](http://BigIdeasMath.com).

### Make a stem-and-leaf plot of the data.

1.

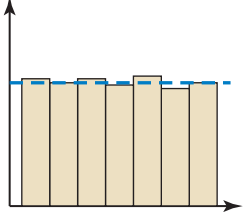
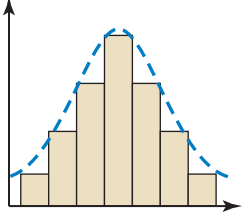
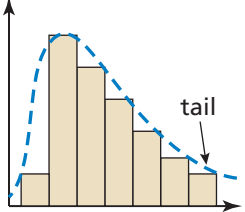
Weights of Dogs (pounds)					
33	33	55	44	39	26
34	42	52	34	48	58
28	39	42	40	45	62
56	26	49	37	64	23

2.

Test Scores (%)					
85	82	100	82	93	76
84	59	71	89	79	87
91	100	89	78	90	85
75	96	99	86	84	92

# Histograms

A **histogram** is a bar graph that shows the frequency of data values in intervals of the same size. The height of a bar represents the frequency of the values in the interval. You can use a histogram to identify the shape of a distribution.

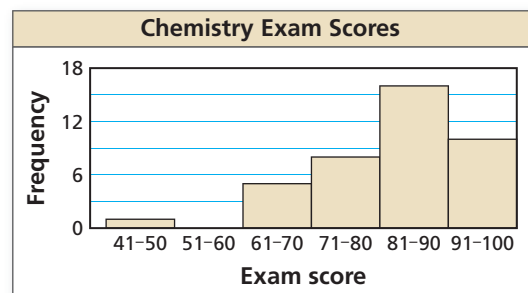
Uniform	Symmetric	Skewed
 <p>All the bars are about the same height. A uniform distribution is also symmetric</p>	 <p>The data on the right of the distribution are approximately a mirror image of the data on the left of the distribution.</p>	 <p>The “tail” extends either left or right. A distribution is <i>skewed left</i> when most of the data are on the right and <i>skewed right</i> when most of the data are on the left.</p>

**Example 1** The frequency table shows the chemistry exam scores for a class. Display the data in a histogram. Describe the distribution.

Score	41–50	51–60	61–70	71–80	81–90	91–100
Frequency	1	0	5	8	16	10

Draw and label the axes. Then draw a bar to represent the frequency of each interval. There is no space between the bars of a histogram. Be sure to include the interval 51–60 with a frequency of 0. The bar height is 0.

► Most of the data are on the right and the tail of the graph extends to the left. So, the distribution is skewed left.



## Practice

Check your answers at [BigIdeasMath.com](http://BigIdeasMath.com).

Display the data in a histogram.  
Describe the distribution.

1.

Speeds of Vehicles On a Highway					
Speed (mi/h)	35–44	45–54	55–64	65–74	75–84
Frequency	2	15	30	18	3

2.

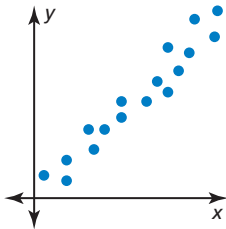
Ages of People at a Family Picnic					
Age	0–19	20–39	40–59	60–79	80–99
Frequency	10	14	9	6	3

# Scatter Plots

A **scatter plot** is a graph that shows the relationship between two data sets. The two data sets are graphed as ordered pairs in a coordinate plane. Scatter plots can show trends in the data.

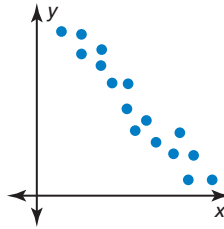
A **correlation** is a relationship between data sets. You can use a scatter plot to describe the correlation between data.

### Positive Correlation



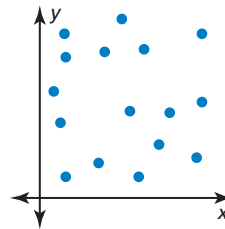
As  $x$  increases,  $y$  increases.

### Negative Correlation



As  $x$  increases,  $y$  decreases.

### No Correlation



The points show no pattern.

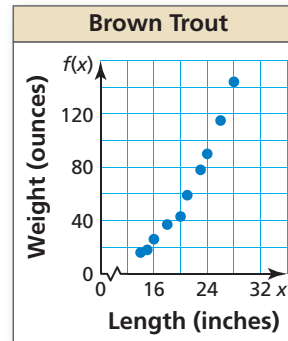
**Example 1** The table shows the lengths and weights of 10 brown trout. Create a scatter plot of the data. Tell whether the data show a *positive*, a *negative*, or *no* correlation.

<b>Length (inches), <math>x</math></b>	16	20	18	15	14	26	21	28	23	24
<b>Weight (ounces), <math>f(x)</math></b>	26	43	37	18	16	115	59	144	78	90

Plot the ordered pairs in a coordinate plane.

As the length increases, the weight increases.

► So, the scatter plot shows a positive correlation.



## Practice

Check your answers at [BigIdeasMath.com](http://BigIdeasMath.com).

Create a scatter plot of the data. Tell whether the data show a *positive*, a *negative*, or *no* correlation.

1.

<b>Height (inches), <math>x</math></b>	72	68	64	70	67	65
<b>Weight (pounds), <math>f(x)</math></b>	180	162	118	145	143	174

2.

<b>Temperature (°F), <math>x</math></b>	38	80	54	76	31	46
<b>Bowls of warm soup, <math>f(x)</math></b>	44	20	32	22	46	38