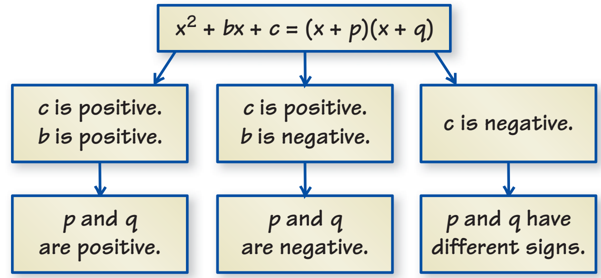
Name ______________________________ Date ______________

Factoring Polynomials

Writing a polynomial as a product of factors is called *factoring*. To factor $x^2 + bx + c$ as $(x + p)(x + q)$, find p and q such that $p + q = b$ and $pq = c$. The diagram shows the relationships between the signs of b and c and the signs of p and q.

To factor $ax^2 + bx + c$, where $a \neq 1$, look for the GCF of the terms of the polynomial, and then factor further if possible. You can also use special product patterns to factor polynomials.



Perfect Square Trinomial Pattern	Difference of Two Squares Pattern
$a^2 + 2ab + b^2 = (a + b)^2$ $a^2 - 2ab + b^2 = (a - b)^2$	$a^2 - b^2 = (a + b)(a - b)$

Example 1 **Factor each polynomial.**

a. $2x^2 + 6x$ **b.** $4x^2 - 25$ **c.** $x^2 + 9x + 18$ **d.** $4x^2 - 21x + 5$

a. The GCF of 2 and 6 is 2. The GCF of x^2 and x is x. So, the greatest common monomial factor of the terms is $2x$.

▶ So, $2x^2 + 6x = 2x(x + 3)$.

b. Use the difference of two squares pattern.

$$4x^2 - 25 = (2x)^2 - 5^2$$
$$= (2x + 5)(2x - 5)$$

c. Notice that $a = 1$, $b = 9$, and $c = 18$. Because b and c are positive, p and q are positive. Find two positive integer factors of 18 whose sum is 9.

▶ So, $x^2 + 9x + 18 = (x + 3)(x + 6)$.

Factors of 18	1, 18	2, 9	3, 6
Sum of factors	19	11	9

The values of p and q are 3 and 6.

d. Notice that $a = 4$, $b = -21$, $c = 5$, and there is no GCF. Because b is negative and c is positive, both factors of c must be negative.

▶ So, $4x^2 - 21x + 5 = (x - 5)(4x - 1)$.

Factors of 4	Factors of 5	Possible factorization	Middle term	
1, 4	−1, −5	$(x - 1)(4x - 5)$	$-9x$	✗
1, 4	−5, −1	$(x - 5)(4x - 1)$	$-21x$	✓
2, 2	−1, −5	$(2x - 1)(2x - 5)$	$-12x$	✗

Practice

Check your answers at BigIdeasMath.com.

Factor the polynomial.

1. $8x - 2$ **2.** $10x^2 + 5x$ **3.** $25x - 10y$ **4.** $x^2 - 7x + 12$

5. $x^2 - x - 20$ **6.** $3x^2 + 6x - 24$ **7.** $4x^2 + 9x + 5$ **8.** $-18x^2 - 6x + 4$

9. $x^2 - 9$ **10.** $8x^2 - 50$ **11.** $x^2 + 14x + 49$ **12.** $3x^2 - 12x + 12$

Name ______________________________ Date ______________

Zeros of Quadratic Functions

A **zero of a function** f is an x-value for which $f(x) = 0$. If a real number k is a zero of the function $f(x) = ax^2 + bx + c$, then k is an x-intercept of the graph of the function.

Example 1 **Find the zeros of each function.**

a. $f(x) = 9x^2 - 1$

Set $f(x)$ equal to 0. Then use square roots to solve for x.

$$9x^2 - 36 = 0$$
$$9x^2 = 36$$
$$x^2 = 4$$
$$x = \pm\sqrt{4}$$
$$x = \pm 2$$

▶ The zeros of the function are $x = -2$ and $x = 2$.

b. $f(x) = x^2 - 2x - 8$

Set $f(x)$ equal to 0. Then use factoring to solve for x.

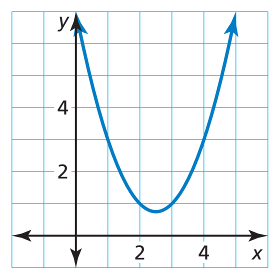
$$x^2 - 2x - 8 = 0$$
$$(x - 4)(x + 2) = 0$$
$$x - 4 = 0 \quad \text{or} \quad x + 2 = 0$$
$$x = 4 \quad \text{or} \quad x = -2$$

▶ The zeros of the function are $x = -2$ and $x = 4$.

Example 2 **Find the zeros of $f(x) = x^2 - 5x + 7$.**

Set $f(x)$ equal to 0. Then use the Quadratic Formula to solve for x.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(7)}}{2(1)}$$
$$= \frac{5 \pm \sqrt{-3}}{2}$$
$$= \frac{5 \pm i\sqrt{3}}{2}$$



▶ The zeros of the function are $x = \frac{5}{2} + \frac{\sqrt{3}}{2}i$ and $x = \frac{5}{2} - \frac{\sqrt{3}}{2}i$.

Notice that the graph of f does not intersect the x-axis.

Practice

Check your answers at BigIdeasMath.com.

Find the zero(s) of the function.

1. $f(x) = 8x^2 + 32$

2. $f(x) = -5x^2 + 40$

3. $f(x) = x^2 - 8x + 16$

4. $f(x) = 4x^2 + 12x + 9$

5. $f(x) = 4(x + 5)(x - 1)$

6. $f(x) = -\frac{1}{2}x(x + 3)$

7. $f(x) = 3x^2 + 12x + 15$

8. $f(x) = 2x^2 - x - 15$

9. $f(x) = -(x + 1)^2 + 18$

10. $f(x) = (x - 7)^2 + 9$

Name ______________________________ Date ______________

Properties of Exponents

Product of Powers	Power of a Product	Power of a Power	
$a^m \cdot a^n = a^{m+n}$ Add exponents.	$(ab)^m = a^m b^m$ Find the power of each factor.	$(a^m)^n = a^{mn}$ Multiply exponents.	
Quotient of Powers	**Power of a Quotient**	**Negative Exponent**	**Zero Exponent**
$\frac{a^m}{a^n} = a^{m-n}, a \neq 0$ Subtract exponents.	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}, b \neq 0$ Find the power of the numerator and the power of the denominator.	$a^{-n} = \frac{1}{a^n}, a \neq 0$	$a^0 = 1, a \neq 0$

Example 1 **Evaluate (a) 4.9^0 and (b) $(-3)^{-4}$.**

a. $4.9^0 = 1$ Definition of zero exponent

b. $(-3)^{-4} = \frac{1}{(-3)^4}$ Definition of negative exponent

$= \frac{1}{81}$ Evaluate power.

Example 2 **Simplify each expression. Write your answer using only positive exponents.**

a. $2^3 \cdot 2^4 = 2^7 = 128$

b. $\frac{5^9}{5^6} = 5^{9-6} = 5^3 = 125$

c. $\frac{12y^0}{x^{-7}} = 12y^0x^7 = 12x^7$

d. $\frac{x^6 \cdot x^2}{x^5} = \frac{x^{6+2}}{x^5} = x^{8-5} = x^3$

e. $(z^4)^2 = z^{4 \cdot 2} = z^8$

f. $(6mn)^3 = 6^3 \cdot m^3 \cdot n^3 = 216m^3n^3$

g. $\left(\frac{y}{3}\right)^4 = \frac{y^4}{3^4} = \frac{y^4}{81}$

h. $\frac{10x^6y^{-2}}{5x^3y} = \frac{10}{5}x^{(6-3)}y^{(-2-1)} = 2x^3y^{-3} = \frac{2x^3}{y^3}$

Practice

Check your answers at BigIdeasMath.com.

Evaluate the expression.

1. $(-9)^0$

2. -8^{-1}

3. 4^{-3}

4. $\frac{-5^0}{3^{-2}}$

Simplify the expression. Write your answer using only positive exponents.

5. $2^9 \cdot 2^{-6}$

6. $-\frac{10^8}{10^{12}}$

7. $y \cdot y^{-5}$

8. $\frac{x^7}{x^{-7}}$

9. $-5x^7 \cdot x^{-11} \cdot 2x^4$

10. $\frac{x^{-2}}{5z^0}$

11. $(w^2)^{-3}$

12. $(8xy)^2$

13. $3x^5 \cdot (-2x)^4$

14. $(-5m^2n^{-1})^3$

15. $\frac{z^8}{z^{-2} \cdot z^9}$

16. $\frac{(x^5)^3}{x^6}$

17. $\left(\frac{3x}{2}\right)^3$

18. $\left(\frac{6x^4}{5y}\right)^{-2}$

19. $\frac{xy^{-2}}{x^4y^{-3}}$

20. $\frac{8xy}{6x^5yz^{-2}}$

21. **METRIC SYSTEM** There are 10^6 micrometers in a meter and 10^3 meters in a kilometer. How many micrometers are there in 10^6 kilometers?

Name ______________________________ Date ______________

Adding and Subtracting Fractions

To add or subtract two fractions with *like denominators*, write the sum or difference of the numerators over the denominator.

Adding or Subtracting Fractions with Like Denominators

$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$, where $c \neq 0$ $\qquad$ $\frac{a}{c} - \frac{b}{c} = \frac{a-b}{c}$, where $c \neq 0$

Example 1 Find $\frac{7}{12} + \frac{1}{12}$.

$\frac{7}{12} + \frac{1}{12} = \frac{7+1}{12}$ Add the numerators.

$= \frac{8}{12}$, or $\frac{2}{3}$ Simplify.

Example 2 Find $\frac{7}{9} - \frac{2}{9}$.

$\frac{7}{9} - \frac{2}{9} = \frac{7-2}{9}$ Subtract the numerators.

$= \frac{5}{9}$ Simplify.

To add or subtract two fractions with *unlike denominators*, first write equivalent fractions with a common denominator. There are two methods you can use.

Adding or Subtracting Fractions with Unlike Denominators

Method 1 Multiply the numerator and the denominator of each fraction by the denominator of the other fraction.

Method 2 Use the **least common denominator** (LCD). The LCD of two or more fractions is the least common multiple (LCM) of the denominators.

Example 3 Find $\frac{1}{8} + \frac{5}{6}$.

Method 1: $\frac{1}{8} + \frac{5}{6} = \frac{1 \cdot 6}{8 \cdot 6} + \frac{5 \cdot 8}{6 \cdot 8}$ Rewrite using a common denominator of $8 \cdot 6 = 48$.

$= \frac{6}{48} + \frac{40}{48}$ Multiply.

$= \frac{46}{48}$, or $\frac{23}{24}$ Simplify.

Example 4 Find $5\frac{3}{4} - 1\frac{7}{10}$.

Method 2: Rewrite the difference as $\frac{23}{4} - \frac{17}{10}$.

The LCM of 4 and 10 is 20. So, the LCD is 20.

$\frac{23}{4} - \frac{17}{10} = \frac{23 \cdot 5}{4 \cdot 5} - \frac{17 \cdot 2}{10 \cdot 2}$ Rewrite using the LCD, 20.

$= \frac{115}{20} - \frac{34}{20}$ Multiply.

$= \frac{81}{20}$, or $4\frac{1}{20}$ Simplify.

Practice

Check your answers at BigIdeasMath.com.

Evaluate.

1. $\frac{1}{14} + \frac{5}{14}$ **2.** $\frac{2}{5} + \frac{1}{5}$ **3.** $\frac{9}{10} - \frac{1}{10}$ **4.** $\frac{11}{16} - \frac{3}{16}$

5. $\frac{5}{8} + \frac{7}{8}$ **6.** $\frac{1}{6} + \frac{1}{6}$ **7.** $\frac{7}{9} + \frac{2}{3}$ **8.** $\frac{3}{5} + \frac{4}{7}$

9. $\frac{3}{4} - \frac{1}{6}$ **10.** $\frac{7}{12} - \frac{5}{9}$ **11.** $\frac{9}{10} - \frac{5}{6}$ **12.** $\frac{5}{12} + \frac{11}{16}$

13. $2\frac{3}{5} + 1\frac{2}{5}$ **14.** $4\frac{6}{7} - 2\frac{4}{7}$ **15.** $5\frac{5}{12} + 3\frac{3}{8}$

16. $8\frac{1}{3} - 3\frac{2}{11}$ **17.** $\frac{1}{2} + 3\frac{2}{9}$ **18.** $4\frac{3}{14} - \frac{1}{7}$

19. $\frac{2}{7} + \frac{3}{4} + \frac{1}{2}$ **20.** $\frac{13}{16} - \frac{1}{4} - \frac{3}{8}$ **21.** $2\frac{1}{6} - \frac{5}{9} + \frac{2}{3}$

Name ______________________________ Date ______________

Multiplying and Dividing Fractions

To multiply two fractions, multiply the numerators and multiply the denominators.

Multiplying Fractions
$\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}$, where b, $d \neq 0$

Example 1 **Find $\frac{2}{5} \cdot \frac{3}{8}$.**

$\frac{2}{5} \cdot \frac{3}{8} = \frac{2 \cdot 3}{5 \cdot 8}$ Multiply the numerators. Multiply the denominators.

$= \frac{\overset{1}{\cancel{2}} \cdot 3}{8 \cdot \cancel{8}_{4}}$ Divide out common factors.

$= \frac{3}{20}$ Simplify.

Example 2 **Find $5\frac{1}{2} \cdot \frac{3}{4}$.**

$5\frac{1}{2} \cdot \frac{3}{4} = \frac{11}{2} \cdot \frac{3}{4}$ Rewrite $5\frac{1}{2}$ as $\frac{11}{2}$.

$= \frac{11 \cdot 3}{2 \cdot 4}$ Multiply the numerators. Multiply the denominators.

$= \frac{33}{8}$, or $4\frac{1}{8}$ Simplify.

Two numbers whose product is 1 are **reciprocals**. To write the reciprocal of a number, write the number as a fraction. Then invert the fraction. Every number except 0 has a reciprocal.

To divide a number by a fraction, multiply the number by the reciprocal of the fraction.

Dividing Fractions
$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{a \cdot d}{b \cdot c}$, where b, c, $d \neq 0$

Example 3 **Find $\frac{3}{7} \div \frac{5}{6}$.**

$\frac{3}{7} \div \frac{5}{6} = \frac{3}{7} \cdot \frac{6}{5}$ Multiply by the reciprocal of $\frac{5}{6}$, which is $\frac{6}{5}$.

$= \frac{3 \cdot 6}{7 \cdot 5}$ Multiply.

$= \frac{18}{35}$ Simplify.

Example 4 **Find $8 \div 2\frac{1}{3}$.**

$8 \div 2\frac{1}{3} = 8 \div \frac{7}{3}$ Rewrite $2\frac{1}{3}$ as $\frac{7}{3}$.

$= 8 \cdot \frac{3}{7}$ Multiply by the reciprocal of $\frac{7}{3}$, which is $\frac{3}{7}$.

$= \frac{8 \cdot 3}{7}$ Multiply.

$= \frac{24}{7}$, or $3\frac{3}{7}$ Simplify.

Practice

Check your answers at BigIdeasMath.com.

Write the reciprocal of the number.

1. $\frac{3}{8}$ **2.** 7 **3.** -12 **4.** $-\frac{6}{5}$

Evaluate.

5. $\frac{3}{4} \cdot \frac{1}{6}$ **6.** $\frac{3}{10} \cdot \frac{2}{3}$ **7.** $\frac{4}{9} \cdot \frac{2}{9}$ **8.** $\frac{5}{8} \cdot \frac{7}{12}$

9. $4 \cdot \frac{3}{16}$ **10.** $3\frac{1}{2} \cdot \frac{6}{7}$ **11.** $1\frac{7}{20} \cdot 2\frac{4}{5}$ **12.** $\frac{1}{10} \cdot 10$

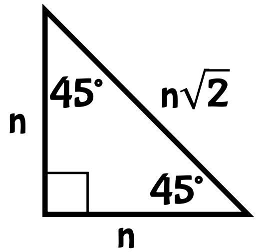
13. $\frac{1}{6} \div \frac{1}{2}$ **14.** $\frac{7}{8} \div \frac{7}{8}$ **15.** $\frac{9}{10} \div \frac{3}{5}$ **16.** $\frac{3}{4} \div \frac{5}{8}$

17. $18 \div \frac{2}{3}$ **18.** $7\frac{1}{2} \div 2\frac{1}{10}$ **19.** $6\frac{3}{7} \div 3$ **20.** $1\frac{3}{25} \div \frac{1}{5}$

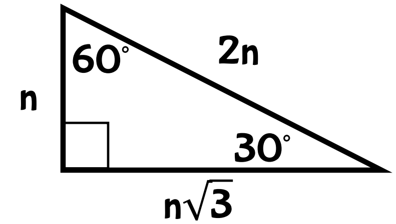
21. **AREA** Find the area of a rectangular court that is $21\frac{3}{5}$ meters long and $13\frac{3}{4}$ meters wide.

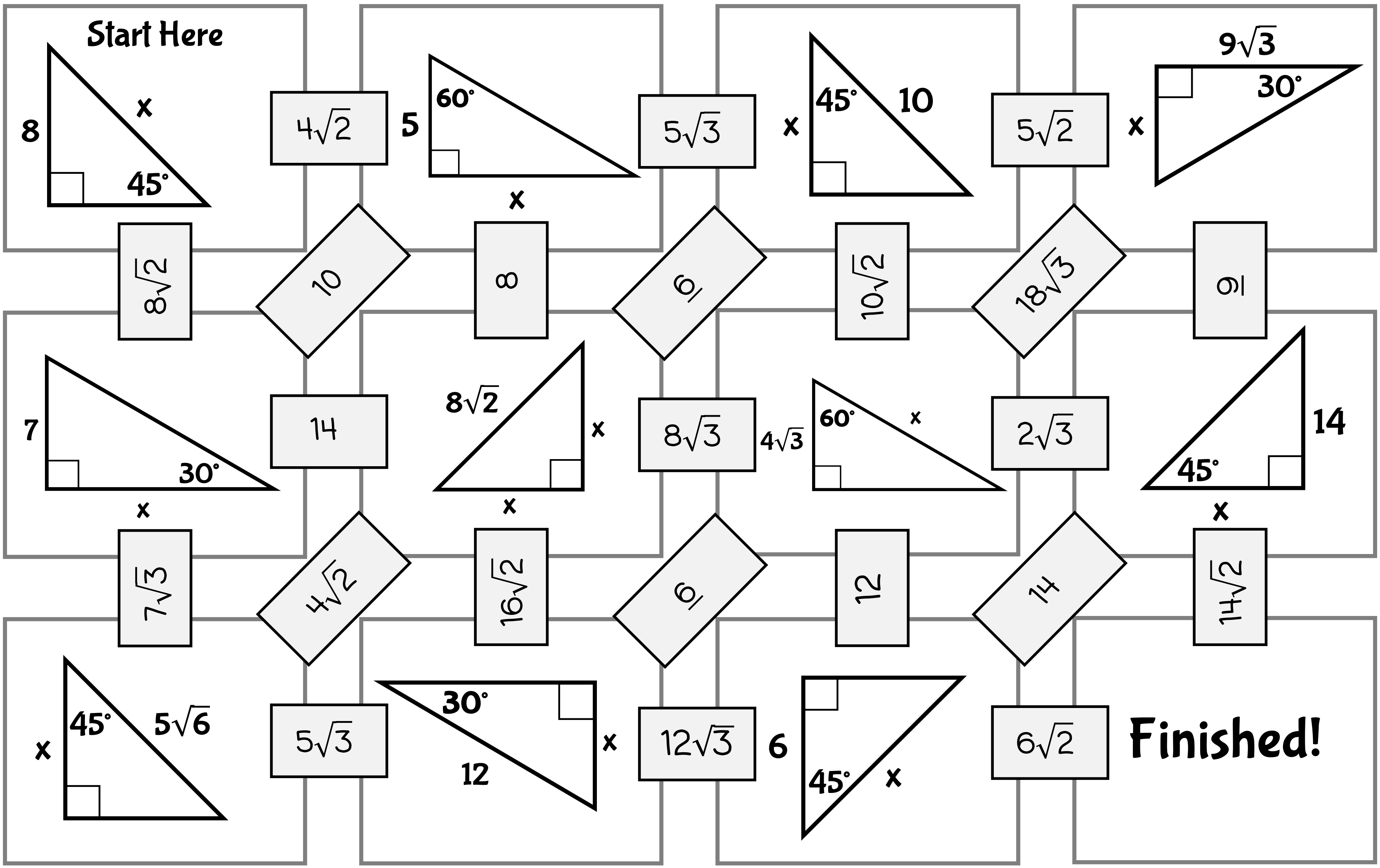
22. **CARPENTRY** How many $1\frac{1}{4}$-foot pieces can you cut from a piece of wood that is 20 feet long?

Special Right Triangle Maze



Start at the "Start Here" box. <u>Solve for x</u>, then follow the path with the correct answer. You have completed the maze when you reach the "Finished" box.





GEOMETRY REFERENCE SHEET

COORDINATE FORMULAS

Given Points: $A(x_1, y_1)$, $B(x_2, y_2)$

Distance: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Slope: $m = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1}$

Midpoint: $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

SYMBOLS + MEANINGS

$\angle$	Angle	$\cong$	Congruent
$\overset{\frown}{AB}$	Arc AB	$^\circ$	Degree
$m\overset{\frown}{AB}$	Measure of arc AB	$\parallel$	Parallel
$\overleftrightarrow{AB}$	Line AB	$\perp$	Perpendicular
$\overrightarrow{AB}$	Ray AB	$\sim$	Similar
$\overline{AB}$	Line segment AB	$\triangle$	Triangle
AB	Length of line segment AB		

VOLUME FORMULAS

pyramid: $V = \frac{1}{3}\pi r^2 h$

cone: $V = \frac{1}{3}Bh$

sphere: $V = \frac{4}{3}r^3$

cylinder: $V = \pi r^2 h$

prism: $V = Bh$

AREA FORMULAS

triangle: $A = \frac{1}{2}bh$

rectangle: $A = lw$

trapezoid: $A = \frac{1}{2}h(b_1 + b_2)$

parallelogram: $A = bh$

circle: $A = \pi r^2$

TRIANGLE CONGRUENCE

SSS SAS ASA AAS HL

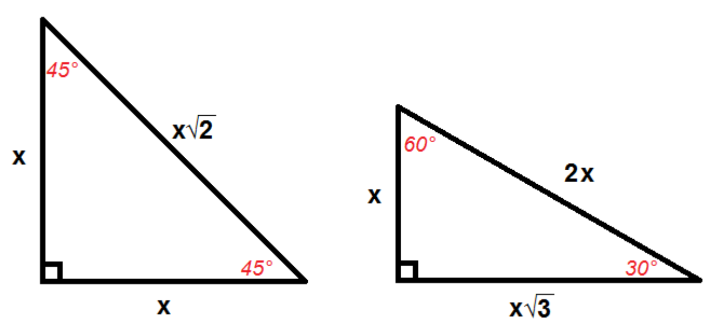
TRIANGLE SIMILARITY

SSS SAS AA

$1^3 = 1$
$2^3 = 8$
$3^3 = 27$
$4^3 = 64$
$5^3 = 125$
$6^3 = 216$
$7^3 = 343$
$8^3 = 512$
$9^3 = 729$
$10^3 = 1000$
$11^3 = 1331$
$12^3 = 1728$

$\sqrt{4} = 2$
$\sqrt{9} = 3$
$\sqrt{16} = 4$
$\sqrt{25} = 5$
$\sqrt{36} = 6$
$\sqrt{49} = 7$
$\sqrt{64} = 8$
$\sqrt{81} = 9$
$\sqrt{100} = 10$
$\sqrt{121} = 11$
$\sqrt{144} = 12$
$\sqrt{169} = 13$
$\sqrt{196} = 14$
$\sqrt{225} = 15$
$\sqrt{256} = 16$
$\sqrt{289} = 17$
$\sqrt{324} = 18$
$\sqrt{361} = 19$

SPECIAL RIGHT TRIANGLES

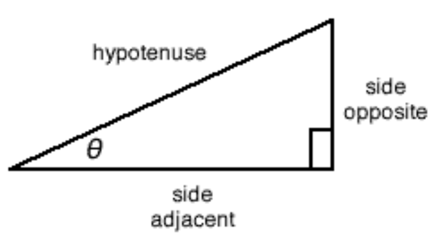


TRIGONOMETRIC RATIOS

sin $\frac{opposite}{hypotenuse}$ $s = \frac{o}{h}$ SOH

cos $\frac{adjacent}{hypotenuse}$ $c = \frac{a}{h}$ CAH

tan $\frac{opposite}{adjacent}$ $t = \frac{o}{a}$ TOA



75%	3/4	0.75
66.66%	2/3	0.66
50%	1/2	0.50
33.33%	1/3	0.33
25%	1/4	0.25
20%	1/5	0.20
12.5%	1/8	0.125
10%	1/10	0.10
5%	1/20	0.05