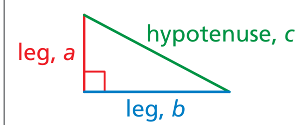
Name ______________________________ Date ______________

The Pythagorean Theorem

In a right triangle, the **hypotenuse** is the side opposite the right angle. The **legs** are the two sides that form the right angle.

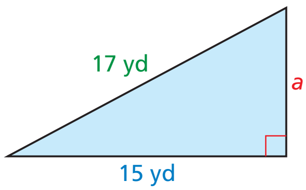
The **Pythagorean Theorem** states that in any right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse.

Pythagorean Theorem



$a^2 + b^2 = c^2$

Example 1 **Find the missing length of the triangle.**

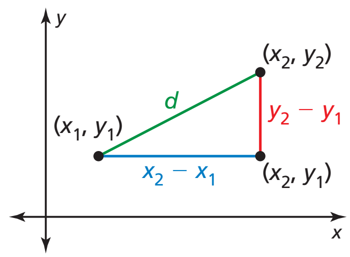


$a^2 + b^2 = c^2$	Write the Pythagorean Theorem.
$a^2 + 15^2 = 17^2$	Substitute 15 for *b* and 17 for *c*.
$a^2 + 225 = 289$	Evaluate powers.
$a^2 = 64$	Subtract 225 from each side.
$a = 8$	Take positive square root of each side.

▶ The missing length is 8 yards.

You can use the Pythagorean Theorem to develop the *Distance Formula*. You can use the **Distance Formula** to find the distance d between any two points (x_1, y_1) and (x_2, y_2) in a coordinate plane.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



Example 2 **Find the distance between the two points.**

a. $(3, 6), (-2, 4)$

Let $(x_1, y_1) = (3, 6)$ and $(x_2, y_2) = (-2, 4)$.

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(-2 - 3)^2 + (4 - 6)^2} \\ &= \sqrt{25 + 4} \\ &= \sqrt{29} \end{aligned}$$

b. $(0, 5), (4, -1)$

Let $(x_1, y_1) = (0, 5)$ and $(x_2, y_2) = (4, -1)$.

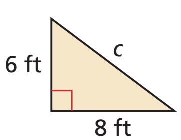
$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(4 - 0)^2 + (-1 - 5)^2} \\ &= \sqrt{16 + 36} \\ &= 2\sqrt{13} \end{aligned}$$

Practice

Check your answers at BigIdeasMath.com.

Find the missing length of the triangle.

1.



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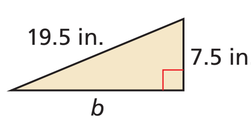

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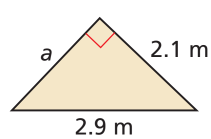


Find the distance between the two points.

4. $(0, 0), (4, 3)$

5. $(0, -7), (5, 5)$

6. $(4, 2), (-1, 5)$

7. $(-5, 6), (-7, -2)$

8. $(-1, -3), (9, 0)$

9. $(-4, -4), (-1, -1)$

Name ______________________________________ Date ______________

Solving Proportions

In the proportion $\frac{a}{b} = \frac{c}{d}$, the products $a \cdot d$ and $b \cdot c$ are called **cross products**. To solve proportions, use the Cross Products Property.

Cross Products Property	
Words The cross products of a proportion are equal.	
Numbers	**Algebra**
$\frac{2}{3} = \frac{4}{6}$	$\frac{a}{b} = \frac{c}{d}$
$2 \cdot 6 = 3 \cdot 4$	$ad = bc$, where $b \neq 0$ and $d \neq 0$

Example 1 **Solve each proportion.**

a. $\frac{x}{6} = \frac{5}{2}$

$x \cdot 2 = 6 \cdot 5$ Cross Products Property

$2x = 30$ Multiply.

$x = 15$ Divide.

▶ The solution is 15.

b. $\frac{8}{y} = \frac{4}{9}$

$8 \cdot 9 = y \cdot 4$

$72 = 4y$

$18 = y$

▶ The solution is 18.

Practice

Check your answers at BigIdeasMath.com.

Solve the proportion.

1. $\frac{1}{3} = \frac{x}{6}$

2. $\frac{2}{5} = \frac{y}{10}$

3. $\frac{z}{9} = \frac{2}{3}$

4. $\frac{2}{7} = \frac{j}{14}$

5. $\frac{4}{9} = \frac{k}{36}$

6. $\frac{m}{24} = \frac{3}{8}$

7. $\frac{11}{3} = \frac{p}{6}$

8. $\frac{n}{54} = \frac{8}{3}$

9. $\frac{14}{a} = \frac{7}{2}$

10. $\frac{15}{b} = \frac{3}{5}$

11. $\frac{21}{2} = \frac{42}{d}$

12. $\frac{9}{16} = \frac{27}{g}$

13. $\frac{21}{r} = \frac{7}{5}$

14. $\frac{25}{q} = \frac{5}{2}$

15. $\frac{9}{8} = \frac{36}{s}$

16. $\frac{4}{15} = \frac{20}{t}$

17. $\frac{x}{2.4} = \frac{3.1}{1.2}$

18. $\frac{4.8}{1.5} = \frac{m}{4.5}$

19. $\frac{3.3}{y} = \frac{1.1}{1.6}$

20. $\frac{2.8}{5.4} = \frac{1.4}{c}$

21. **PENCILS** Thirty-six pencils are packaged in 6 boxes. How many pencils are packaged in 10 boxes?

22. **TICKETS** Two tickets cost \$15. How much does it cost to buy seven tickets?

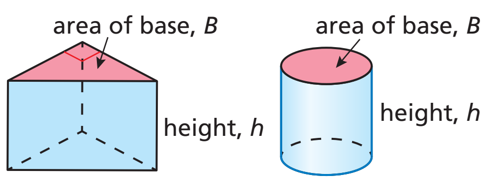
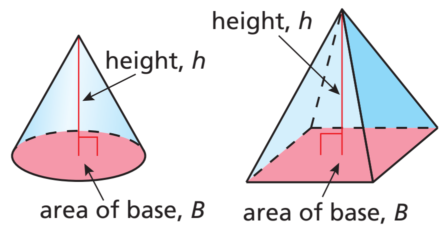
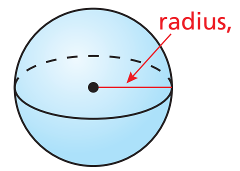
23. **SALADS** Three salads cost \$6.50. How much does it cost to buy six salads?

24. **FIELD TRIP** There are 108 students on a field trip. The ratio of girls to boys is 5 to 4. How many are girls?

Name ______________________________ Date ______________

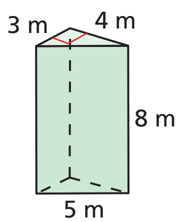
Volume

A **volume** of a solid is a measure of the amount of space that it occupies. Volume is measured in *cubic units*. You can use the following formulas to find volumes.

Prism and Cylinder	Cone and Pyramid	Sphere
		
$V = Bh$	$V = \frac{1}{3}Bh$	$V = \frac{4}{3}\pi r^3$

Example 1 **Find the volume of each solid.**

a.



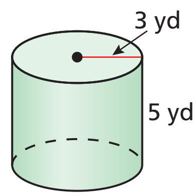
$V = Bh$

$= \frac{1}{2}(3)(4) \bullet 8$

$= 6 \bullet 8$

$= 48 \text{ m}^3$

b.

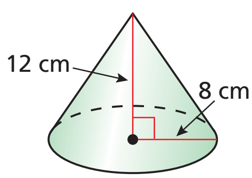


$V = Bh$

$= \pi(3)^2 \bullet 5$

$= 45\pi \approx 141 \text{ yd}^3$

c.

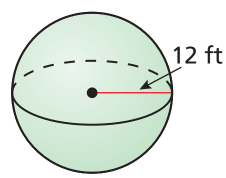


$V = \frac{1}{3}Bh$

$= \frac{1}{3}\pi(8)^2 \bullet 12$

$= 256\pi \approx 804 \text{ cm}^3$

d.



$V = \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi(12)^3$

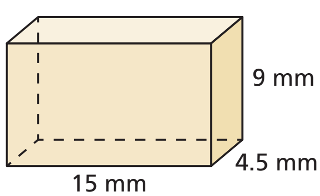
$= 2304\pi \approx 7238 \text{ ft}^3$

Practice

Check your answers at BigIdeasMath.com.

Find the volume of the solid.

1.



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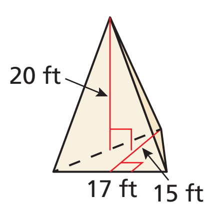

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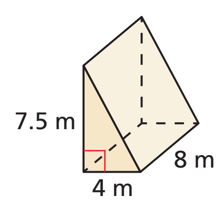

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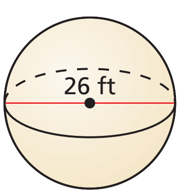

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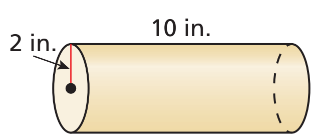

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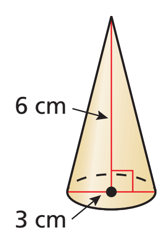


Name ______________________________ Date ______________

Rewriting Literal Equations

An equation that has two or more variables is called a **literal equation**. To rewrite a literal equation, solve for one variable in terms of the other variable(s).

Example 1 **Solve each literal equation for y.**

a.

$3x + 5y = 45$	
$3x - 3x + 5y = 45 - 3x$	Subtract $3x$ from each side.
$5y = 45 - 3x$	Simplify.
$\frac{5y}{5} = \frac{45 - 3x}{5}$	Divide each side by 5.
$y = 9 - \frac{3}{5}x$	Simplify.

▶ The rewritten literal equation is $y = 9 - \frac{3}{5}x$.

b.

$2xy + 5y = 7$	
$y(2x + 5) = 7$	Distributive Property
$\frac{y(2x + 5)}{2x + 5} = \frac{7}{2x + 5}$	Divide each side by $2x + 5$.
$y = \frac{7}{2x + 5}$	Simplify.

▶ The rewritten literal equation is $y = \frac{7}{2x + 5}$.

c.

$2x = \frac{3 + y}{y}$	
$2x \cdot y = \frac{3 + y}{y} \cdot y$	Multiply each side by y.
$2xy = 3 + y$	Simplify.
$2xy - y = 3 + y - y$	Subtract y from each side.
$2xy - y = 3$	Simplify.
$y(2x - 1) = 3$	Distributive Property
$\frac{y(2x - 1)}{2x - 1} = \frac{3}{2x - 1}$	Divide each side by $2x - 1$.
$y = \frac{3}{2x - 1}$	Simplify.

▶ The rewritten literal equation is $y = \frac{3}{2x - 1}$.

Practice

Check your answers at BigIdeasMath.com.

Solve the literal equation for y.

1. $x + 3y = 9$

2. $4x - 2y = 16$

3. $2x + 7y = 5$

4. $2x + 3y = 6$

5. $5x - 4y = 10$

6. $x - 2y = 8$

7. $2xy - 6 = 8x$

8. $4x = 9y + xy$

9. $4yz = 3y - 8x$

10. $2xy = 3z + 4y$

11. $\frac{2 + 7y}{y} = x$

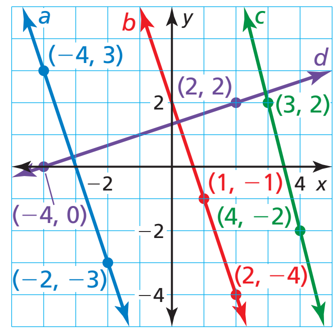
12. $3x = \frac{5 + y}{y}$

Name ______________________ Date ____________

Parallel and Perpendicular Lines

Parallel lines are coplanar lines that do not intersect. Nonvertical parallel lines have the same slope. Two lines that intersect to form a right angle are **perpendicular lines**. Two nonvertical lines are perpendicular if and only if the product of their slopes is -1.

Example 1 **Determine which of the lines are parallel and which are perpendicular.**



Find the slope of each line.

Line a: $m = \dfrac{3-(-3)}{-4-(-2)} = -3$

Line b: $m = \dfrac{-1-(-4)}{1-2} = -3$

Line c: $m = \dfrac{2-(-2)}{3-4} = -4$

Line d: $m = \dfrac{2-0}{2-(-4)} = \dfrac{1}{3}$

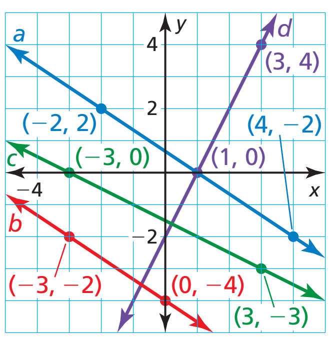
▶ Because lines a and b have the same slope, lines a and b are parallel. Because $\frac{1}{3}(-3) = -1$, lines a and d are perpendicular and lines b and d are perpendicular.

Practice

Check your answers at BigIdeasMath.com.

Determine which of the lines are parallel and which are perpendicular.

1.



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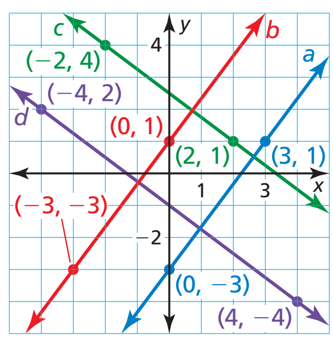

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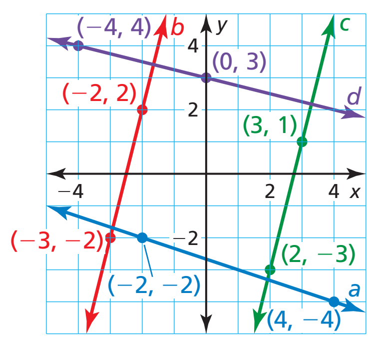


4. **GEOMETRY** The vertices of a quadrilateral are $A(-5, 3)$, $B(2, 2)$, $C(4, -3)$, and $D(-2, -2)$. Is the quadrilateral a parallelogram? Explain your reasoning.

5. **GEOMETRY** The vertices of a parallelogram are $J(-5, 0)$, $K(1, 4)$, $L(3, 1)$, and $M(-3, -3)$. Is the parallelogram a rectangle? Explain your reasoning.

GEOMETRY REFERENCE SHEET

COORDINATE FORMULAS

Given Points: $A(x_1, y_1)$, $B(x_2, y_2)$

Distance: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Slope: $m = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1}$

Midpoint: $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

SYMBOLS + MEANINGS

$\angle$	Angle	$\cong$	Congruent
$\overset{\frown}{AB}$	Arc *AB*	$^\circ$	Degree
$m\overset{\frown}{AB}$	Measure of arc *AB*	$\parallel$	Parallel
$\overleftrightarrow{AB}$	Line *AB*	$\perp$	Perpendicular
$\overrightarrow{AB}$	Ray *AB*	$\sim$	Similar
$\overline{AB}$	Line segment *AB*	$\triangle$	Triangle
AB	Length of line segment *AB*		

VOLUME FORMULAS

pyramid: $V = \frac{1}{3}\pi r^2 h$

cone: $V = \frac{1}{3}Bh$

sphere: $V = \frac{4}{3}r^3$

cylinder: $V = \pi r^2 h$

prism: $V = Bh$

AREA FORMULAS

triangle: $A = \frac{1}{2}bh$

rectangle: $A = lw$

trapezoid: $A = \frac{1}{2}h(b_1 + b_2)$

parallelogram: $A = bh$

circle: $A = \pi r^2$

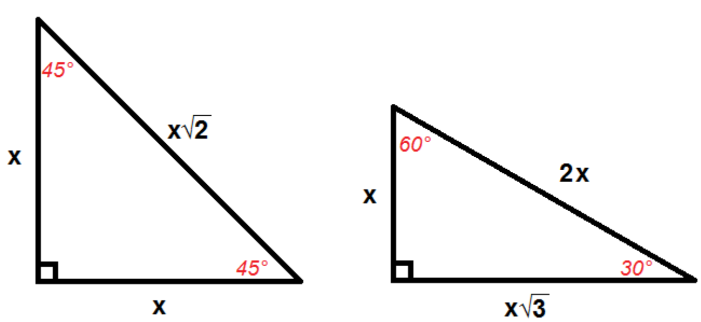
TRIANGLE CONGRUENCE

SSS SAS ASA AAS HL

TRIANGLE SIMILARITY

SSS SAS AA

SPECIAL RIGHT TRIANGLES



$1^3 = 1$
$2^3 = 8$
$3^3 = 27$
$4^3 = 64$
$5^3 = 125$
$6^3 = 216$
$7^3 = 343$
$8^3 = 512$
$9^3 = 729$
$10^3 = 1000$
$11^3 = 1331$
$12^3 = 1728$

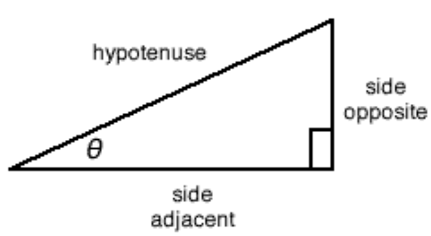
$\sqrt{4} = 2$
$\sqrt{9} = 3$
$\sqrt{16} = 4$
$\sqrt{25} = 5$
$\sqrt{36} = 6$
$\sqrt{49} = 7$
$\sqrt{64} = 8$
$\sqrt{81} = 9$
$\sqrt{100} = 10$
$\sqrt{121} = 11$
$\sqrt{144} = 12$
$\sqrt{169} = 13$
$\sqrt{196} = 14$
$\sqrt{225} = 15$
$\sqrt{256} = 16$
$\sqrt{289} = 17$
$\sqrt{324} = 18$
$\sqrt{361} = 19$

TRIGONOMETRIC RATIOS

sin $\frac{opposite}{hypotenuse}$ $s = \frac{o}{h}$ SOH

cos $\frac{adjacent}{hypotenuse}$ $c = \frac{a}{h}$ CAH

tan $\frac{opposite}{adjacent}$ $t = \frac{o}{a}$ TOA



75%	3/4	0.75
66.66%	2/3	0.66
50%	1/2	0.50
33.33%	1/3	0.33
25%	1/4	0.25
20%	1/5	0.20
12.5%	1/8	0.125
10%	1/10	0.10
5%	1/20	0.05