

Scientific Notation

A number is written in **scientific notation** when it is represented as the product of a factor and a power of 10. The factor must be greater than or equal to 1 and less than 10.

$$c \times 10^n \qquad 1 \leq c < 10 \text{ and } n \text{ is an integer}$$

When a number is written in scientific notation, you can write the number in standard form using the absolute value of the exponent n . When n is negative, move the decimal point $|n|$ places to the left. When n is positive, move the decimal point $|n|$ places to the right.

Example 1 Write (a) 7.4×10^5 and (b) 3.96×10^{-4} in standard form.

a. $7.4 \times 10^5 = 740,000$ Move decimal point $|5| = 5$ places to the right.



b. $3.96 \times 10^{-4} = 0.000396$ Move decimal point $|-4| = 4$ places to the left.



When a number is written in standard form, you can write the number in scientific notation using the following steps.

Step 1 Move the decimal point so it is located to the right of the leading nonzero digit.

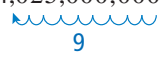
Step 2 Count the number n of places you moved the decimal point. The exponent of the power of 10 is n when you move the decimal point to the left and $-n$ when you move the decimal point to the right.

Example 2 Write each number in scientific notation.

a. 4,025,000,000

$$4,025,000,000 = 4.025 \times 10^9$$


Move decimal point 9 places to the left. The exponent is 9.



b. 0.00000591

$$0.00000591 = 5.91 \times 10^{-6}$$

Move decimal point 6 places to the right. The exponent is -6 .



Practice

Check your answers at BigIdeasMath.com.

Write the number in standard form.

- | | | | |
|-------------------------|--------------------------|------------------------|--------------------------|
| 1. 2×10^4 | 2. 8.4×10^1 | 3. 7×10^{-3} | 4. 5.05×10^{-1} |
| 5. 1.8×10^{-7} | 6. 6.29×10^{-5} | 7. 5.591×10^0 | 8. 3.0504×10^9 |

Write the number in scientific notation.

- | | | | |
|-----------------|-------------|-----------------|-------------|
| 9. 400 | 10. 72,000 | 11. 0.8 | 12. 0.00019 |
| 13. 100,500,000 | 14. 324,900 | 15. 0.000002621 | 16. 0.05008 |

17. **NEURONS** A study found that the average human brain has 8.6×10^{10} neurons. Write this number in standard form. The cell bodies of neurons have widths between 0.000004 meter and 0.0001 meter. Write this range in scientific notation.

Solving Linear Equations

To determine whether a value is a solution of an equation, substitute the value into the equation and simplify.

Example 1 Determine whether (a) $x = 1$ or (b) $x = -2$ is a solution of $5x - 1 = 4$.

a. $5x - 1 = -2x + 6$

$$5(1) - 1 \stackrel{?}{=} -2(1) + 6 \quad \text{Substitute.}$$

$$4 = 4 \quad \checkmark \quad \text{Simplify.}$$

► So, $x = 1$ is a solution.

b. $5x - 1 = -2x + 6$

$$5(-2) - 1 \stackrel{?}{=} -2(-2) + 6 \quad \text{Substitute.}$$

$$-11 \neq 10 \quad \times \quad \text{Simplify.}$$

► So, $x = -2$ is *not* a solution.

To solve a linear equation, isolate the variable.

Example 2 Solve each equation. Check your solution.

a. $4x - 3 = 13$

$$4x - 3 + 3 = 13 + 3 \quad \text{Add 3.}$$

$$4x = 16 \quad \text{Simplify.}$$

$$\frac{4x}{4} = \frac{16}{4} \quad \text{Divide by 4.}$$

$$x = 4 \quad \text{Simplify.}$$

Check

$$4x - 3 = 13$$

$$4(4) - 3 \stackrel{?}{=} 13$$

$$13 = 13 \quad \checkmark$$

b. $2(y - 8) = y + 6$

$$2y - 16 = y + 6 \quad \text{Distributive Property}$$

$$2y - y - 16 = y - y + 6 \quad \text{Subtract } y.$$

$$y - 16 = 6 \quad \text{Simplify.}$$

$$y - 16 + 16 = 6 + 16 \quad \text{Add 16.}$$

$$y = 22 \quad \text{Simplify.}$$

Check

$$2(y - 8) = y + 6$$

$$2(22 - 8) \stackrel{?}{=} 22 + 6$$

$$28 = 28 \quad \checkmark$$

Practice

Check your answers at BigIdeasMath.com.

Determine whether (a) $x = -1$ or (b) $x = 3$ is a solution of the equation.

1. $5x + 7 = 2$

2. $-4x + 8 = -4$

3. $2x - 1 = 3x - 4$

Solve the equation. Check your solution.

4. $x - 9 = 24$

5. $n + 14 = 0$

6. $-16 = 4y$

7. $-\frac{5}{6}t = -15$

8. $81 = 46 - x$

9. $4x + 5 = 1$

10. $x + 5 = 11x$

11. $9(y - 3) = 45$

12. $6 = 7k + 8 - k$

13. $6n + 3 = -4n + 7$

14. $2c + 5 = 3(c - 8)$

15. $18m + 3(2m + 8) = 0$

16. $\frac{w - 6}{5} = 8$

17. $\frac{15 + h}{3} = 10$

18. $\frac{8 - 3x}{5} = x$

19. $(8r + 6) + (4r - 1) = 14$

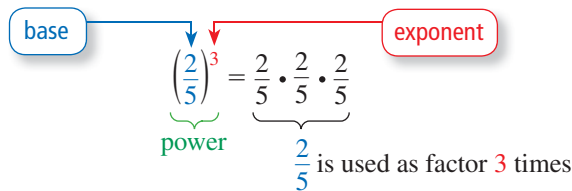
20. $\frac{2}{3}y - 3 = 9$

21. $\frac{1}{2}x - \frac{3}{10} = \frac{5}{2}x + \frac{7}{10}$

22. **MONEY** You have a total of \$3.25 in change made up of 25 pennies, 6 nickels, 2 dimes, and x quarters. How many quarters do you have?

Powers and Exponents

A **power** is a product of repeated factors. The **base** of a power is the common factor. The **exponent** of a power indicates the number of times the base is used as a factor.



Example 1 Write each product using exponents.

a. $(-9) \cdot (-9) \cdot (-9) \cdot (-9) \cdot (-9)$

Because -9 is used as a factor 5 times, its exponent is 5.

▶ So, $(-9) \cdot (-9) \cdot (-9) \cdot (-9) \cdot (-9) = (-9)^5$.

b. $\pi \cdot \pi \cdot h \cdot h \cdot h$

Because π is used as a factor 2 times, its exponent is 2. Because h is used as a factor 3 times, its exponent is 3.

▶ So, $\pi \cdot \pi \cdot h \cdot h \cdot h = \pi^2 h^3$.

Example 2 Evaluate each expression.

a. $(-5)^4$

$$\begin{aligned} (-5)^4 &= (-5) \cdot (-5) \cdot (-5) \cdot (-5) \\ &= 625 \end{aligned}$$

Write as repeated multiplication.

Simplify.

b. -5^4

$$\begin{aligned} -5^4 &= -(5 \cdot 5 \cdot 5 \cdot 5) \\ &= -625 \end{aligned}$$

Write as repeated multiplication.

Simplify.

Practice

Check your answers at BigIdeasMath.com.

Write the product using exponents.

1. $7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7$

2. $\left(-\frac{1}{3}\right) \cdot \left(-\frac{1}{3}\right) \cdot \left(-\frac{1}{3}\right)$

3. $x \cdot x \cdot y \cdot y \cdot y \cdot y \cdot y$

4. $2.5 \cdot 2.5 \cdot b \cdot b \cdot b \cdot b$

5. $(-n) \cdot (-n) \cdot (-n) \cdot (-n)$

6. $(-12) \cdot (-12) \cdot v \cdot v \cdot v$

Evaluate the expression.

7. 10^4

8. -15^2

9. $\left(\frac{3}{4}\right)^3$

10. $\left(-\frac{1}{2}\right)^5$

11. **VOLUME** Write an expression involving a power that represents the volume (in cubic centimeters) of the die shown. Then find the volume.



Simplifying Algebraic Expressions

Parts of an algebraic expression are called *terms*. **Like terms** are terms that have the same variables raised to the same exponents. Constant terms are also like terms.

An algebraic expression is in **simplest form** when it has no like terms and no parentheses. To *combine* like terms that have variables, use the Distributive Property to add or subtract the coefficients.

Example 1 Simplify $8y + 7y$.

$$\begin{aligned} 8y + 7y &= (8 + 7)y \\ &= 15y \end{aligned}$$

Distributive Property

Add coefficients.

Example 2 Simplify $2(x + 5) - 3(x - 2)$.

$$\begin{aligned} 2(x + 5) - 3(x - 2) &= 2(x) + 2(5) - 3(x) - 3(-2) \\ &= 2x + 10 - 3x + 6 \\ &= 2x - 3x + 10 + 6 \\ &= -x + 16 \end{aligned}$$

Distributive Property

Multiply.

Group like terms.

Combine like terms.

Example 3 Simplify $xy + 3y - 2x + 5y - 3xy$.

$$\begin{aligned} xy + 3y - 2x + 5y - 3xy &= xy - 3xy + 3y + 5y - 2x \\ &= -2xy + 8y - 2x \end{aligned}$$

Group like terms.

Combine like terms.

Practice

Check your answers at BigIdeasMath.com.

Simplify the expression.

1. $7x + 15x$

2. $8y - 14y$

3. $7d + 9 - 5d$

4. $3w + 2(2 - 3w) + 2$

5. $(x + 3) + (3x - 7)$

6. $(5k + 6) + (4k - 8)$

7. $(-7n + 6) + (5n + 15)$

8. $(9z + 12) - (6z + 8)$

9. $(8b + 1) - (-10b - 5)$

10. $s(8 - 2t) + 3t(4 - 2s) + 5t$

11. $qr + 2q^2 - 3qr - r^2 - 6q^2$

12. $g^3(h - 4g) - h(3 - 2g^3)$

13. **EARNINGS** The original price of a model car is d dollars. You use a coupon and buy the kit for $(d - 10)$ dollars. You assemble the model car and sell it for $(2d - 20)$ dollars. Write an expression that represents your earnings. Interpret the expression.

Solving Systems of Equations

A **system of linear equations** is a set of two or more linear equations in the same variables. An example is shown at the right. A **solution of a system of linear equations** in two variables is an ordered pair that is a solution of each equation in the system.

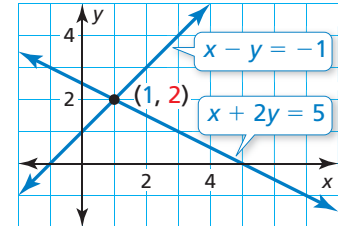
$x + 2y = 5$	Equation 1
$x - y = -1$	Equation 2

Example 1 Solve the system of linear equations above by (a) graphing, (b) substitution, and (c) elimination.

a. Graph each equation. The graphs appear to intersect at (1, 2). Check this point.

Equation 1 $x + 2y = 5$
 $1 + 2(2) \stackrel{?}{=} 5$
 $5 = 5$ ✓

Equation 2 $x - y = -1$
 $1 - 2 \stackrel{?}{=} -1$
 $-1 = -1$ ✓



▶ The solution is (1, 2).

b. Solve for x in Equation 2. $x - y = -1$
 $x = y - 1$

Substitute $y - 1$ for x in Equation 1 and solve for y . $x + 2y = 5$
 $(y - 1) + 2y = 5$
 $y = 2$

Substitute 2 for y in Equation 2 and solve for x . $x - y = -1$
 $x - 2 = -1$
 $x = 1$

▶ The solution is (1, 2).

c. Multiply Equation 2 by 2. Then add the equations and solve the resulting equation.

$$\begin{array}{r} x + 2y = 5 \Rightarrow x + 2y = 5 \\ x - y = -1 \Rightarrow 2x - 2y = -2 \\ \hline 3x = 3 \\ x = 1 \end{array}$$

Substitute 1 for x in Equation 2 and solve for y . $x - y = -1$
 $1 - y = -1$
 $2 = y$

▶ The solution is (1, 2).

Practice

Check your answers at BigIdeasMath.com.

Solve the system of linear equations by graphing.

- | | | | |
|--------------------------------|----------------------------------|----------------------------------|----------------------------------|
| 1. $y = x - 3$
$y = -x + 1$ | 2. $-x + 2y = -1$
$x + y = 4$ | 3. $2x + y = 5$
$4x - 2y = 6$ | 4. $9x - 3y = 3$
$3x + y = 1$ |
|--------------------------------|----------------------------------|----------------------------------|----------------------------------|

Solve the system of linear equations by substitution.

- | | | | |
|---------------------------------|--------------------------------|----------------------------------|------------------------------------|
| 5. $y = 1 - x$
$-2x + y = 4$ | 6. $x = y + 3$
$5x - y = 7$ | 7. $3x - y = 5$
$2x - y = -3$ | 8. $x - 2y = -3$
$7x - 2y = 15$ |
|---------------------------------|--------------------------------|----------------------------------|------------------------------------|

Solve the system of linear equations by elimination.

- | | | | |
|------------------------------------|-----------------------------------|------------------------------------|--------------------------------------|
| 9. $-2x + 2y = -2$
$2x + y = 5$ | 10. $x - 4y = -3$
$4x + y = 5$ | 11. $x + 5y = -2$
$5x + y = 14$ | 12. $2x + 3y = 5$
$4x + 6y = -10$ |
|------------------------------------|-----------------------------------|------------------------------------|--------------------------------------|