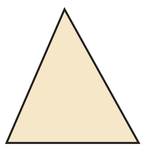
Name ______________________________ Date ______________

Classifying Triangles

You can use angle measures and side lengths to classify triangles.

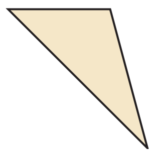
Classifying Triangles Using Angles

acute triangle



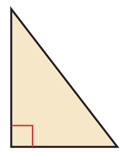
all acute angles

obtuse triangle



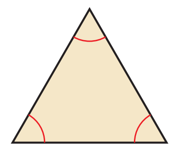
1 obtuse angle

right triangle



1 right angle

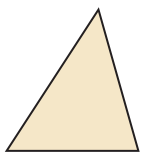
equiangular triangle



3 congruent angles

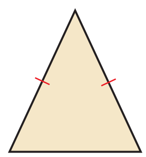
Classifying Triangles Using Sides

scalene triangle



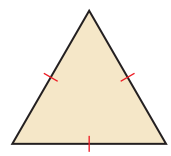
no congruent sides

isosceles triangle



at least 2 congruent sides

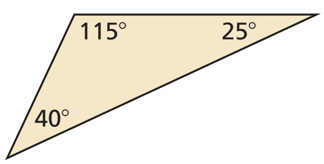
equilateral triangle



3 congruent sides

Example 1 **Classify each triangle by its angles and by its sides.**

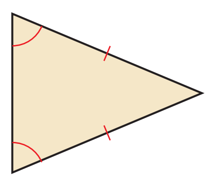
a.



The triangle has one obtuse angle and no congruent sides.

▶ So, the triangle is an obtuse scalene triangle.

b.



The triangle has all acute angles and two congruent sides.

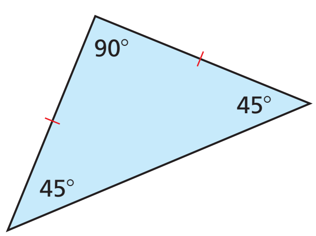
▶ So, the triangle is an acute isosceles triangle.

Practice

Check your answers at BigIdeasMath.com.

Classify the triangle by its angles and by its sides.

1.



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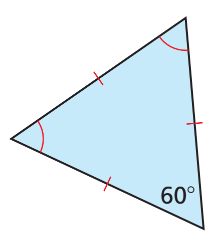

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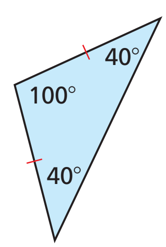

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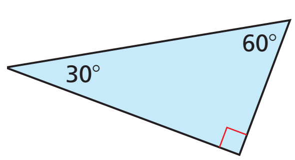

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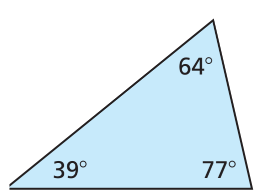

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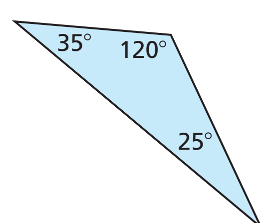


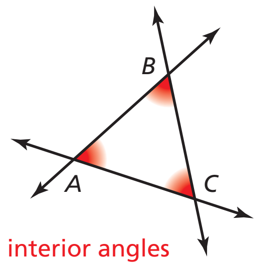
Name ____________________ Date ____________

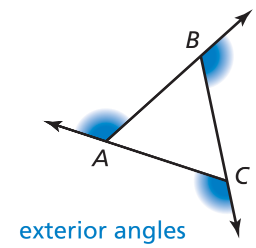
Finding Angles of Triangles

Using Interior and Exterior Angles

When the sides of a polygon are extended, other angles are formed. The original angles are the **interior angles**. The angles that form linear pairs with the interior angles are the **exterior angles**.

The theorems given below show how the angle measures of a triangle are related. You can use these theorems to find angle measures.





Triangle Sum Theorem

The sum of the measures of the interior angles of a triangle is 180°.

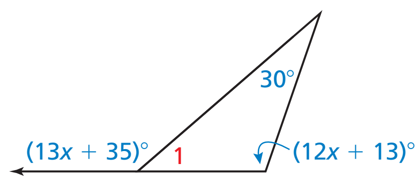
Exterior Angle Theorem

The measure of an exterior angle of a triangle is equal to the sum of the measures of the two nonadjacent interior angles.

Corollary to the Triangle Sum Theorem

The acute angles of a right triangle are complementary.

Example 1 **Find $m\angle 1$.**



First write and solve an equation to find the value of x.

$(13x + 35)° = 30° + (12x + 13)°$ Apply the Exterior Angle Theorem.

$x = 8$ Solve for x.

Substitute 8 for x in $(12x + 13)°$ to find the obtuse angle measure, 109°. Then write and solve an equation to find $m\angle 1$.

$m\angle 1 + 30° + 109° = 180°$ Apply the Triangle Sum Theorem.

$m\angle 1 = 41°$ Solve for $m\angle 1$.

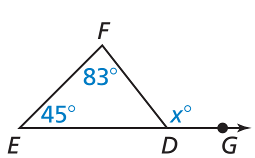
▶ So, the measure of $\angle 1$ is 41°.

Practice

Check your answers at BigIdeasMath.com.

Find the value of x.

1.



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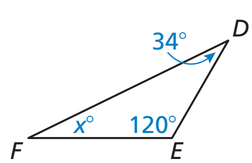

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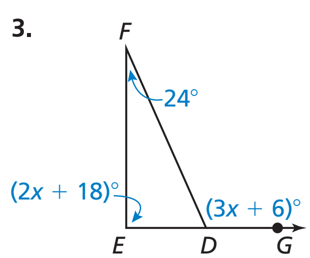

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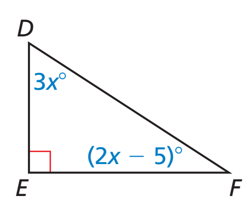
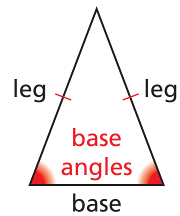


Name ____ Date ________________

Finding Angles of Triangles

Using Isosceles and Equilateral Triangles

When an isosceles triangle has exactly two congruent sides, these two sides are the **legs**. The third side is the **base** of the isosceles triangle. The two angles adjacent to the base are called **base angles**.



You can use the theorems given below to find angle measures and side lengths.

Base Angles Theorem

If two sides of a triangle are congruent, then the angles opposite them are congruent.

Converse of the Base Angles Theorem

If two angles of a triangle are congruent, then the sides opposite them are congruent.

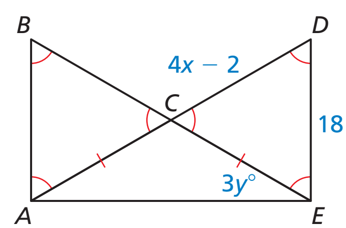
Corollary to the Base Angles Theorem

If a triangle is equilateral, then it is equiangular.

Corollary to the Converse of the Base Angles Theorem

If a triangle is equiangular, then it is equilateral.

Example 1 **Find the values of *x* and *y* in the diagram.**



Step 1 Find the value of x. Because $\triangle CDE$ is equiangular, it is also equilateral by the Corollary to the Converse of the Base Angles Theorem. So, $\overline{CD} \cong \overline{DE}$.

$CD = DE$	Definition of congruent segments
$4x - 2 = 18$	Substitute.
$x = 5$	Solve for x.

Step 2 Find the value of y. By the Triangle Sum Theorem, $3(m\angle DCE) = 180°$, so $m\angle DCE = 60°$. Because $\angle ACE$ and $\angle DCE$ form a linear pair, they are supplementary angles and $m\angle ACE = 180° - 60° = 120°$. The diagram shows that $\triangle ACE$ is isosceles. By the Base Angles Theorem, $\angle CAE \cong \angle CEA$. So, $m\angle CAE = m\angle CEA$.

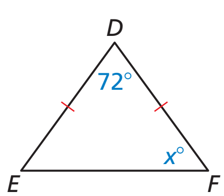
$120° + 3y° + 3y° = 180°$	Apply the Triangle Sum Theorem.
$y = 10$	Solve for y.

Practice

Check your answers at BigIdeasMath.com.

Find the value(s) of the variable(s).

1.



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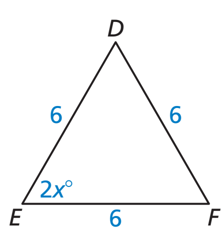

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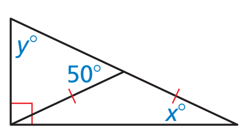

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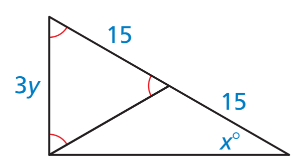


Name ______________________ Date ______________

Parallel Lines and Transversals

Using Properties of Parallel Lines

Corresponding Angles Theorem

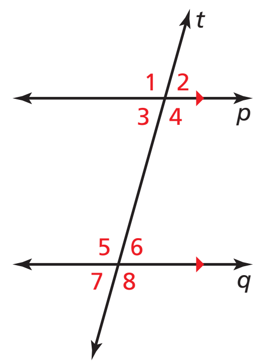
If two parallel lines are cut by a transversal, then the pairs of corresponding angles are congruent.

Examples In the diagram at the right, $\angle 2 \cong \angle 6$ and $\angle 3 \cong \angle 7$.

Alternate Interior Angles Theorem

If two parallel lines are cut by a transversal, then the pairs of alternate interior angles are congruent.

Examples In the diagram at the right, $\angle 3 \cong \angle 6$ and $\angle 4 \cong \angle 5$.



Alternate Exterior Angles Theorem

If two parallel lines are cut by a transversal, then the pairs of alternate exterior angles are congruent.

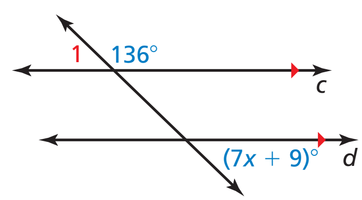
Examples In the diagram at the right, $\angle 1 \cong \angle 8$ and $\angle 2 \cong \angle 7$.

Consecutive Interior Angles Theorem

If two parallel lines are cut by a transversal, then the pairs of consecutive interior angles are supplementary.

Examples In the diagram at the right, $\angle 3$ and $\angle 5$ are supplementary, and $\angle 4$ and $\angle 6$ are supplementary.

Example 1 **Find the value of *x*.**



By the Linear Pair Postulate, $m\angle 1 = 180° - 136° = 44°$. Lines c and d are parallel, so you can use the theorems about parallel lines.

$m\angle 1 = (7x + 9)°$	Alternate Exterior Angles Theorem
$44° = (7x + 9)°$	Substitute 44° for $m\angle 1$.
$35 = 7x$	Subtract 9 from each side.
$5 = x$	Divide each side by 7.

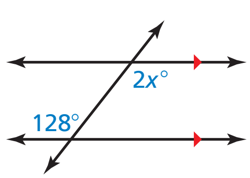
▶ So, the value of x is 5.

Practice

Check your answers at BigIdeasMath.com.

Find the value of *x*.

1.



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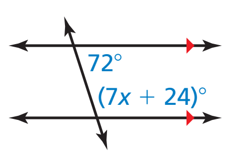

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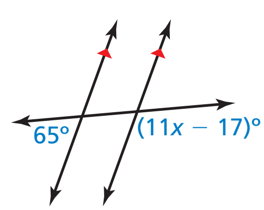

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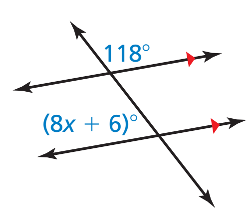


Name ______________________________ Date ____________

Parallel Lines and Transversals

Determining Whether Lines are Parallel

The theorems about angles formed when parallel lines are cut by a transversal have true converses.

Corresponding Angles Converse

If two lines are cut by a transversal so the corresponding angles are congruent, then the lines are parallel.

Alternate Interior Angles Converse

If two lines are cut by a transversal so the alternate interior angles are congruent, then the lines are parallel.

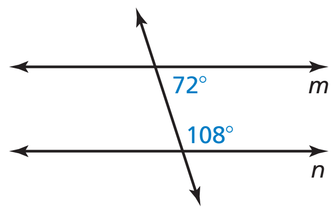
Alternate Exterior Angles Converse

If two lines are cut by a transversal so the alternate exterior angles are congruent, then the lines are parallel.

Consecutive Interior Angles Converse

If two lines are cut by a transversal so the consecutive interior angles are supplementary, then the lines are parallel.

Example 1 **Decide whether there is enough information to prove that $m \parallel n$. If so, state the theorem you would use.**



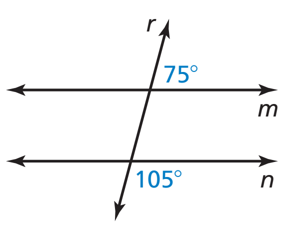
The sum of the marked consecutive interior angles is 180°. Lines m and n are parallel when the consecutive interior angles are supplementary. So, by the Consecutive Interior Angles Converse, $m \parallel n$.

Practice

Check your answers at BigIdeasMath.com.

Decide whether there is enough information to prove that $m \parallel n$. If so, state the theorem you would use.

1.



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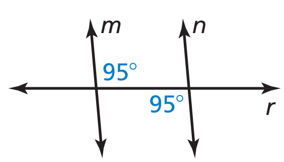

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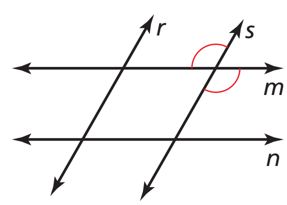

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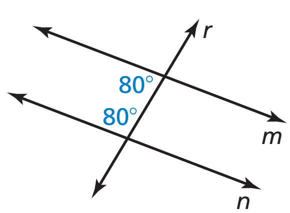

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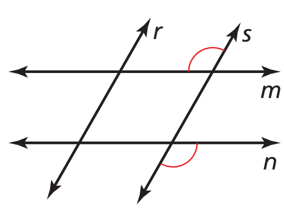

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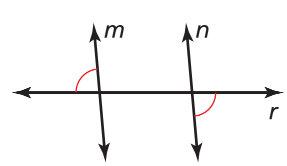


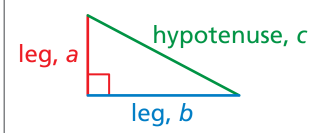
Name ______________________________ Date ______________

The Pythagorean Theorem

In a right triangle, the **hypotenuse** is the side opposite the right angle. The **legs** are the two sides that form the right angle.

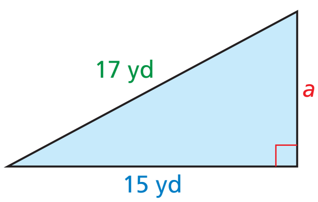
The **Pythagorean Theorem** states that in any right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse.

Pythagorean Theorem



$a^2 + b^2 = c^2$

Example 1 **Find the missing length of the triangle.**

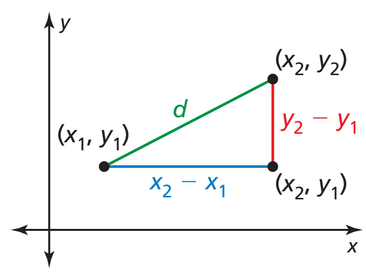


$a^2 + b^2 = c^2$	Write the Pythagorean Theorem.
$a^2 + 15^2 = 17^2$	Substitute 15 for b and 17 for c.
$a^2 + 225 = 289$	Evaluate powers.
$a^2 = 64$	Subtract 225 from each side.
$a = 8$	Take positive square root of each side.

▶ The missing length is 8 yards.

You can use the Pythagorean Theorem to develop the *Distance Formula*. You can use the **Distance Formula** to find the distance d between any two points (x_1, y_1) and (x_2, y_2) in a coordinate plane.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



Example 2 **Find the distance between the two points.**

a. (3, 6), (−2, 4)

Let $(x_1, y_1) = (3, 6)$ and $(x_2, y_2) = (-2, 4)$.

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(-2 - 3)^2 + (4 - 6)^2} \\ &= \sqrt{25 + 4} \\ &= \sqrt{29} \end{aligned}$$

b. (0, 5), (4, −1)

Let $(x_1, y_1) = (0, 5)$ and $(x_2, y_2) = (4, -1)$.

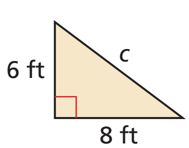
$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(4 - 0)^2 + (-1 - 5)^2} \\ &= \sqrt{16 + 36} \\ &= 2\sqrt{13} \end{aligned}$$

Practice

Check your answers at BigIdeasMath.com.

Find the missing length of the triangle.

1.



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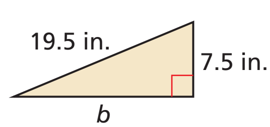
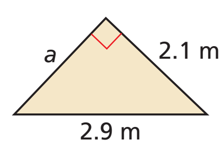


3.



Find the distance between the two points.

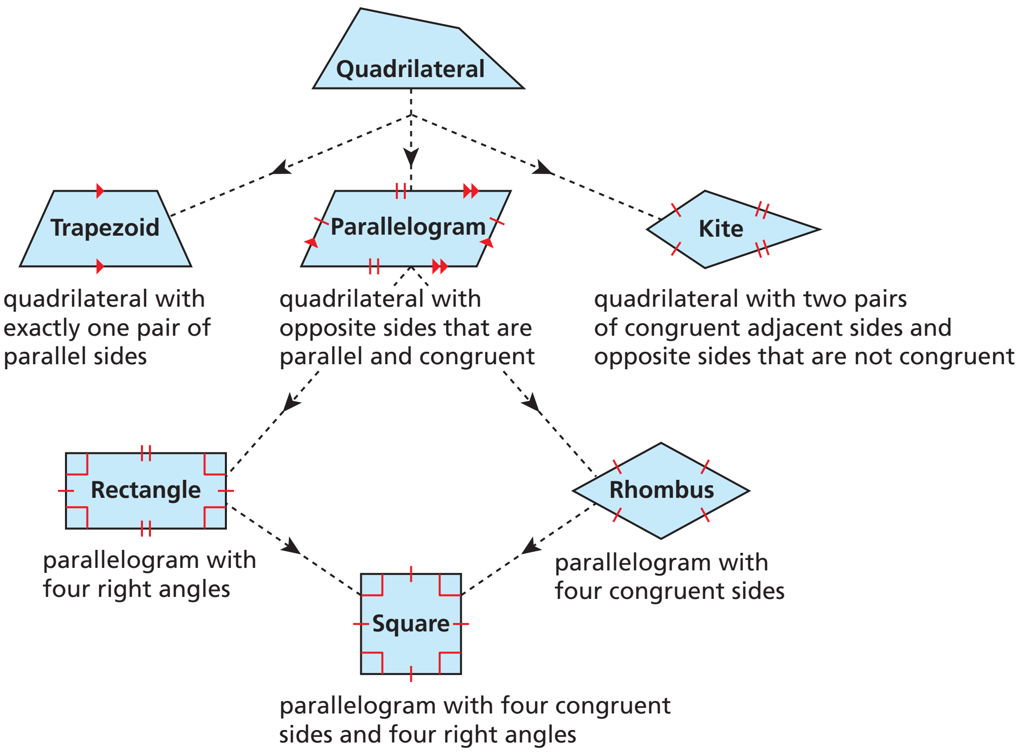
4. (0, 0), (4, 3) **5.** (0, −7), (5, 5) **6.** (4, 2), (−1, 5)

7. (−5, 6), (−7, −2) **8.** (−1, −3), (9, 0) **9.** (−4, −4), (−1, −1)

Name ______________________________ Date ______________

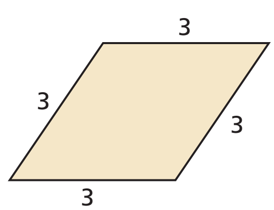
Classifying Quadrilaterals

A **quadrilateral** is a polygon with four sides. The diagram shows properties of different types of quadrilaterals and how they are related. When identifying a quadrilateral, use the name that is most specific.



Example 1 **Classify the quadrilateral.**

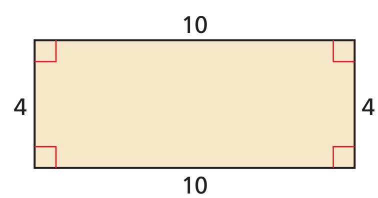
a.



The quadrilateral has four congruent sides.

▶ So, it is a rhombus.

b.



The quadrilateral has four right angles.

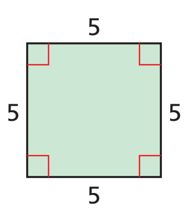
▶ So, it is a rectangle.

Practice

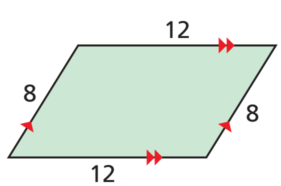
Check your answers at BigIdeasMath.com.

Classify the quadrilateral.

1.



2.



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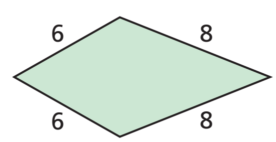
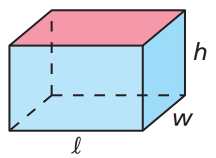
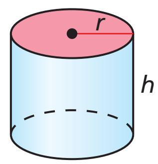
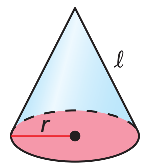
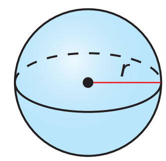


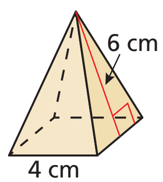
Name ______________________________ Date ______________

Surface Area

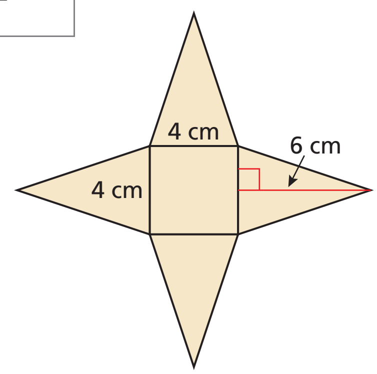
A **solid** is a three-dimensional figure that encloses a space. The **surface area** of a solid is the sum of the areas of all of its faces. Surface area is measured in *square units*. You can use a two-dimensional representation of a solid, called a **net**, to find the surface area of a solid. You can also use the following formulas to find surface areas.

Rectangular Prism	Cylinder	Cone	Sphere
			
$S = 2\ell w + 2\ell h + 2wh$	$S = 2\pi r^2 + 2\pi rh$	$S = \pi r^2 + \pi r\ell$	$S = 4\pi r^2$

Example 1 **Find the surface area of the regular pyramid.**



Draw a net.



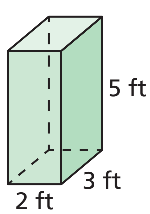
Area of Base $4 \cdot 4 = 16$

Area of a Lateral Face $\frac{1}{2} \cdot 4 \cdot 6 = 12$

▶ There are four identical lateral faces. So, the surface area is $16 + 4(12) = 64$ square centimeters.

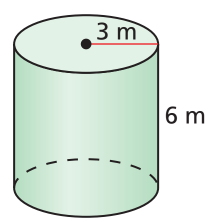
Example 2 **Find the surface area of each solid.**

a.



$$\begin{aligned} S &= 2\ell w + 2\ell h + 2wh \\ &= 2(2)(3) + 2(2)(5) + 2(3)(5) \\ &= 12 + 20 + 30 \\ &= 62 \text{ ft}^2 \end{aligned}$$

b.



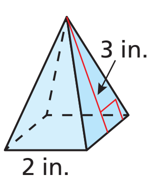
$$\begin{aligned} S &= 2\pi r^2 + 2\pi rh \\ &= 2\pi(3)^2 + 2\pi(3)(6) \\ &= 18\pi + 36\pi \\ &= 54\pi \approx 170 \text{ m}^2 \end{aligned}$$

Practice

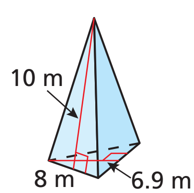
Check your answers at BigIdeasMath.com.

Find the surface area of the regular pyramid.

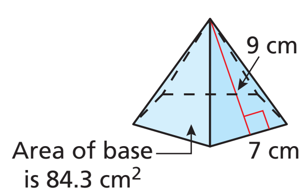
1.



2.

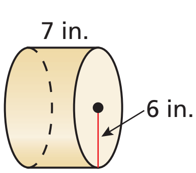


3.

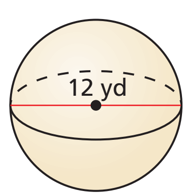


Find the surface area of the solid.

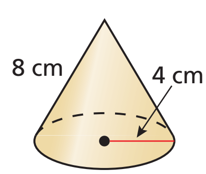
4.



5.



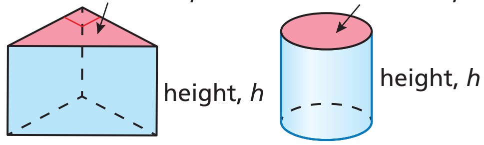
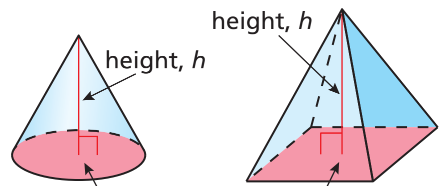
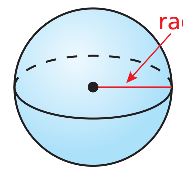
6.



Name ______________________ Date ______________

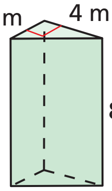
Volume

A **volume** of a solid is a measure of the amount of space that it occupies. Volume is measured in *cubic units*. You can use the following formulas to find volumes.

Prism and Cylinder	Cone and Pyramid	Sphere
		
$V = Bh$	$V = \frac{1}{3}Bh$	$V = \frac{4}{3}\pi r^3$

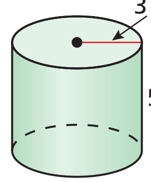
Example 1 **Find the volume of each solid.**

a.



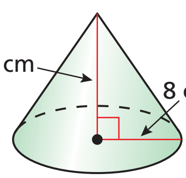
$$V = Bh$$
$$= \tfrac{1}{2}(3)(4) \cdot 8$$
$$= 6 \cdot 8$$
$$= 48 \text{ m}^3$$

b.



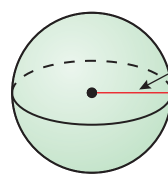
$$V = Bh$$
$$= \pi(3)^2 \cdot 5$$
$$= 45\pi \approx 141 \text{ yd}^3$$

c.



$$V = \tfrac{1}{3}Bh$$
$$= \tfrac{1}{3}\pi(8)^2 \cdot 12$$
$$= 256\pi \approx 804 \text{ cm}^3$$

d.



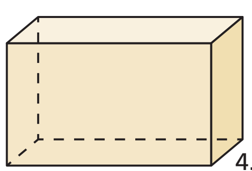
$$V = \tfrac{4}{3}\pi r^3$$
$$= \tfrac{4}{3}\pi(12)^3$$
$$= 2304\pi \approx 7238 \text{ ft}^3$$

Practice

Check your answers at BigIdeasMath.com.

Find the volume of the solid.

1.



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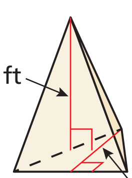
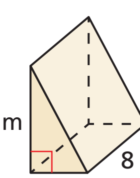
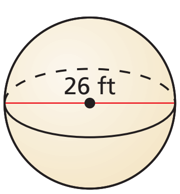


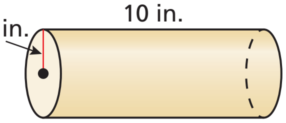
3.



4.



5.



6.