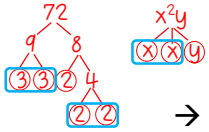
Simplifying Radicals

1. Find the prime factorization of the radicand, including factoring the variables.
2. Find pairs of the same prime factor or variable.
3. Pull out one from each pair, and multiply everything you pull out. Multiply the remaining prime numbers and variables and write that product under the square root sign.

Ex: Simplify $\sqrt{72x^2y}$



→ $3 \cdot 2 \cdot x\sqrt{2 \cdot y}$

→ $6x\sqrt{2y}$

Operations with Radicals

Adding & Subtracting Radicals

1. Simplify the radicals if possible.
2. Add or subtract the numbers in front of like radicals.

Ex: $\sqrt{20} + \sqrt{45}$

→ $2\sqrt{5} + 3\sqrt{5} = 5\sqrt{5}$

Multiplying Radicals

1. Follow the rules for multiplying polynomials (i.e. distributive property, FOIL, etc.). Multiply the numbers in front of the radicals together and multiply the numbers under the radicals together.
2. Simplify the radical if possible.

Ex: $3\sqrt{6} \cdot 4\sqrt{10}$

→ $12\sqrt{60}$ → $12 \cdot 2\sqrt{15} = 24\sqrt{15}$

Dividing Radicals

1. Divide the numbers in front of the radicals and the numbers under the radicals.
2. If you have a radical in the denominator, you need to rationalize the denominator.
   - For a single radical term in the denominator, multiply the numerator and denominator by that same radical.
   - For a two-term denominator containing one or two radicals, multiply the numerator and denominator by the conjugate of the denominator. ($a + b$ and $a - b$ are conjugates)

Ex: $\frac{\sqrt{9}}{\sqrt{18}}$

→ $\frac{\sqrt{1}}{\sqrt{2}}$ → $\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

Ex: $\frac{3}{\sqrt{5}+3}$

$\frac{3}{\sqrt{5}+3} \cdot \frac{(\sqrt{5}-3)}{(\sqrt{5}-3)} = \frac{3\sqrt{5}-9}{5-3\sqrt{5}+3\sqrt{5}-9}$

$= \frac{3\sqrt{5}-9}{-4}$

Radical Equations

1. Isolate the radical if possible.
2. Square both sides of the equation.
3. Solve the equation for the variable. (If it is a quadratic equation, you may find two solutions).
4. CHECK YOUR ANSWER(S) by substituting them in for the variable(s) in the original equation. (Some solutions may end up being extraneous).

Ex: $\sqrt{x+6} - x = 0$

$\sqrt{x+6} - x = 0$ → $\sqrt{x+6} = x$

→ $(\sqrt{x+6})^2 = x^2$

→ $x + 6 = x^2$ → $x^2 - x - 6 = 0$

→ $(x - 3)(x + 2) = 0$ → $x = 3$ or -2

check 3: $\sqrt{3+6} - 3 = 0$ good

check -2: $\sqrt{-2+6} - (-2) \neq 0$ extraneous

→ solution: $x = 3$

Simplify each radical.

119. $\sqrt{90}$	120. $\sqrt{54a^3b^4}$	121. $\sqrt{600x^2y^2z}$

Simplify each radical expression.

122. $\sqrt{18} - \sqrt{50}$	123. $2\sqrt{5}(\sqrt{3} + 8\sqrt{4})$	124. $\frac{5}{\sqrt{3}}$
125. $7\sqrt{3} + 2\sqrt{12} - 3\sqrt{27}$	126. $(\sqrt{2} + 3\sqrt{3}) \cdot (\sqrt{6} - 4\sqrt{2})$	127. $\frac{2}{\sqrt{5} - \sqrt{3}}$

Solve each radical equation.

128. $\sqrt{3x} - 27 = 0$	129. $\sqrt{2x + 11} = \sqrt{6x - 7}$	130. $\sqrt{4x + 1} = x - 1$